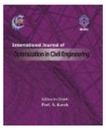
**INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING**
***Int. J. Optim. Civil Eng., 2026; 16(3):325-336***



# PREDICTION OF MODAL STRAIN ENERGY USING MULTI-OBJECTIVE METAHEURISTIC-TRAINED ARTIFICIAL NEURAL NETWORKS

V.R. Mahdavi[1*, †] and A. Kaveh [2]

[1]*Department of Civil and Geomechanics Engineering, Arak University of Technology, Arak, Iran*

[2]*School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran-16, Iran*

## ABSTRACT

This paper uses multi-objective methods to improve the exploration for single-objective problems. This method involved splitting the objective function into two segments and these enhanced using specialized algorithms designed for handling multiple objective functions. Three MO algorithms include Colliding Bodies Optimization (MOCBO), Particle Swarm Optimization (MOPSO), and non-dominated sorting genetic algorithm (NSGA-II), which are used to get the best prediction of structural modal strain energy. The indicated method is then implemented to two spatial truss structures. The recently developed method has superior performance over the previous approach that relied on single-objective optimization (SO) algorithms.

**Keywords:** Neural networks; Strain energy; Optimization algorithms; Multi-objective optimizations; Truss structures.

Received: 20 May 2026; Accepted: 13 July 2026



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

## 1. INTRODUCTION

Researchers frequently analyze modal strain energy of structures in various studies, as it plays a significant role in detecting structural damage [1]. It can be tough and time-intensive to analyze the modal properties of a structure featuring many degrees of freedom. Neural

*Corresponding author: Department of Civil and Geomechanics Engineering, Arak University of Technology, Arak, Iran

†E-mail address: v.r.mahdavi@arakut.ac.ir (V.R. Mahdavi)

networks (NN) belong to the field of machine learning (ML), enabling researchers to utilize advanced computing to develop models that assist in decision-making [2-5]. Thus, using better data-based computer methods is needed to improve how accurately we can make predictions.

Over the past two decades, research on computing strain energy in structures using neural networks has progressed from early feedforward models and handcrafted feature approaches to advanced deep-learning architectures and hybrid physics-informed methods that improve accuracy, generalization, and computational efficiency for complex structural analyses [6, 7].

The enhancement of artificial neural networks is frequently approached from various perspectives: optimization of weights, bias, hyper parameters, network structure, activation nodes, learning parameters, learning algorithm, learning environment, etc. [8]. The geometry properties of sections, the geometry and topology of structures were considered as input variables for training. The output variable utilized was the structural response. This research treats the section areas and strain energy of the structure as the input and output for the artificial neural network, respectively.

This study presents the prediction of strain energy in truss structures using muti-objective optimization algorithms (MOAs). The main contributions of this research work are as follows:

(i) This study introduces an extension of MOA to train the neural networks through numerical examples;
(ii) This problem is solved using the recent MOAs and a comparative study is carried out;
(iii) A comparison is made between the results achieved using MOAs and the traditional strategy based on SOAs.

The present paper is organized as follows: In the next section, the problem formulations of strain energy in structures are provided. The optimization algorithms are then explained, followed by a section consisting of the study of two problems of truss structures. Conclusions are derived in the last section.



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

## 2. PROBLEM FORMULATION

The strain energy of the structure is a very important indicator for structural damage identification. The modal strain energy associated with the *i*th shape mode is given by:

$$\gamma_i = \phi_i K \phi_i \tag{1}$$

in which, $\varphi_i$ is the *i*th shape mode and K is the global stiffness matrix of the structure.
The mathematical equation of a linear regression is expressed as:

$$F(X) = wX + b \tag{2}$$

where F(x) denotes the predicted value, $X$ is the input variable, $w$ and $b$ are the layer weight and bias, respectively.

The problem that was referenced previously was solved using the SO algorithms, and the number of variables matches the total of weight and bias components. By increasing the number of variables, these algorithms prevent finding the best solution, particularly when dealing with structural problems. Then, it is essential to adopt a strategy to enhance the search space exploration in this optimization problem.

In this study, MO algorithms are utilized to train the neural networks. In this method, the objective function is divided into two parts. These objective functions define the differences in the strain energy of structure between the exact and predicted states. Each objective function is utilized to estimate the strain energy associated with a particular shape mode. MO algorithms aim to find weights and biases (*w* and *b*) for the neural network model that simultaneously minimize the objective functions.

The objective functions in MO algorithms must be in conflict with each other, such as when one objective function increases, the other decreases. In this case, the user is given a set of solutions (referred to as the Pareto front) by multi-objective algorithms. The objective functions employed in this research are in accordance with one another, and they are only used to find the optimal point in the problem search space as accurately as possible. The purpose of NN training is to identify a single solution that accurately represents the precise value. Hence, if the multi-objective algorithm only gives one solution, that point is the optimal one. Otherwise, the best solution in the Pareto front will be chosen using the Knee point method described in Reference [9].



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

## 3. OPTIMIZATION ALGORITHMS

As mentioned before, the MOCBO, MOPSO and NSGA-II algorithms have been employed for solving objective functions. In this section, these algorithms are briefly presented.

### *3.1. Multi-Objective Colliding Bodies Optimization algorithm*

Algorithm 1 is named 'multi-objective colliding bodies optimization' (MOCBO) because it utilizes the CBO formulation for the search process. This algorithm aims to find optimal solutions for multiple objective optimization problems by simulating the interactions and collisions between bodies representing potential solutions [10,11].

**Algorithm 1** Multi-objective colliding bodies optimization (MOCBO)

```
Initialize
    Generate an initial population of candidate solutions (bodies)
    Evaluate the objectives for each body
    Initialize the non-dominated archive
Begin
    While (termination condition is met) do
        Evaluate the maximin value of each solution
        Arrange the populations
        Divide the populations into two equal groups
        Determine the collision between bodies
        Update the positions of the bodies
        Update the non-dominated archive
    End While
End
```

### *3.2. Multi-objective Particle Swarm Optimization algorithm*

In the field of multi-objective optimization, MOPSO is a metaheuristic optimization algorithm that has been widely utilized. In the MOPSO a population of candidate solutions called particles iteratively searches for optimal solutions by moving through the search space based on the best solutions so far found [12]. The evaluation of solution quality by using Pareto's dominance is the main concept behind MOPSO. The MOPSO algorithm maintains track of both the global and personal best position for every particle (Algorithm 2).

**Algorithm 2** Multi-objective particle swarm optimization (MOPSO)

```
Initialize
      Initialize the population of particles
      Assign random velocities to each particle
      Evaluate the objectives for each particle
      Initialize the non-dominated archive
Begin
      While (stopping criteria are NOT satisfied) do
          For each particle
              Update the personal best solution (pbest)
              Update the global best solution (gbest)
              Update the velocity and position of each particle
          End For
          Update the non-dominated archive
      End While
End
```

### *3.3 Non-dominated Sorting Genetic Algorithm-II*

The Non-dominated Sorting Genetic Algorithm-II, known as NSGA-II, is a commonly utilized multi-objective optimization technique that aims to discover a set of non-dominated solutions in a given search domain [13]. NSGA-II works by maintaining a population of candidate solutions and evolving them over multiple generations. It uses techniques like non-dominated sorting diversity preservation and elitism to push the search process towards the Pareto-optimal front where no other solution is better among all objectives simultaneously.

In order to preserve diversity of the population, it evaluates individuals according to their dominant relationship with each other and assigns them ranks and crowding distances. By selecting the best individuals through a combination of dominance and diversity measures, NSGA-II is able to achieve an efficient approximation of the Pareto front (Algorithm 3).

**Algorithm 3** Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

```
Initialize
      Generate an initial population of candidate solutions randomly
      Evaluate the objective function values for each individual in the population
      Initialize the non-dominated archive
Begin
      While (termination condition is met) do
          Calculate the crowding distance for individuals in each Pareto front
          Select individuals for the next generation
          Perform crossover and mutation operators on the selected individuals
```



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

Combine the current population and offspring to form a new population
Update the non-dominated archive
**End While**
**End**

## 4. NUMERICAL EXAMPLES

In this section, two spatial truss structures are considered for the numerical study to evaluate the feasibility and effectiveness of the proposed methods. Additionally, the SO algorithms have been used in all examples to compare the outcomes of the implemented method with traditional approaches. In all the examples, a total of 200 populations are used for all MO and SO algorithms. For all examples, the maximum number of iterations is considered as 1000. The programming language used for writing the algorithms is MATLAB.

In both examples, five mode shapes are considered to predict the strain energy of trusses. The first objective function aims to forecast the first to third mode shapes, while the second objective function is designed to estimate the fourth and third mode shapes.

### *4.1. Example 1: A 120-bar dome truss*

The 120-bar dome truss shown in Figure 1 was first analyzed by Soh and Yang to obtain the optimal sizing and configuration variables with stress constraints [14]. The 120 structural members of this spatial truss are categorized as 7 groups using symmetry. Non-structural masses are attached to all free nodes as follows: 3000 *kg* at node one, 500 *kg* at nodes 2–13 and 100 *kg* at the remaining nodes. The material density is taken as $7971.810 kg/m^3$ and the modulus of elasticity is $210{,}000\ MPa$. The range of cross-sectional areas varies from 1 to 129.3 $cm^2$.

Figure 2 shows the trade-off between the objective functions using three multi-objective optimization methods. Since the PF sets have more than one solution, the best solution is chosen using the Knee point method described in Ref. [9]. The best solutions found using MO algorithms are also displayed in Figure 2 with a star symbol. The PF set obtained through MOCBO is dominated by those obtained through MOPSO and NSGA-II, as can be observed. Table 1 displays the best solutions found in this example using SO and MO algorithms. Figure 3 also illustrates the most accurate predictions of the modal strain energy in different modes using MOCBO algorithm. By comparison, one can see that the predicted modal strain energy using MOCBO is closer to the exact value compared to the outcome of the NSGA-II and MOPSO.

### *4.2. Example 2: A 72-bar space truss*

Figure 4 shows the topology and element numbering of a 72-bar space truss for this example. The elements are classified into 16 design groups, as shown in this figure. For this example, the material density is 2770 kg/m$^3$ and the modulus of elasticity is 69,800 MPa. Four non-structural masses of 2270 kg are attached to the nodes 1-4 [15,16]. The range of cross-sectional areas varies from 0.645 to 10 $cm^2$.


[ DOI: 10.22068/ijoce.2026.16.3.680 ]

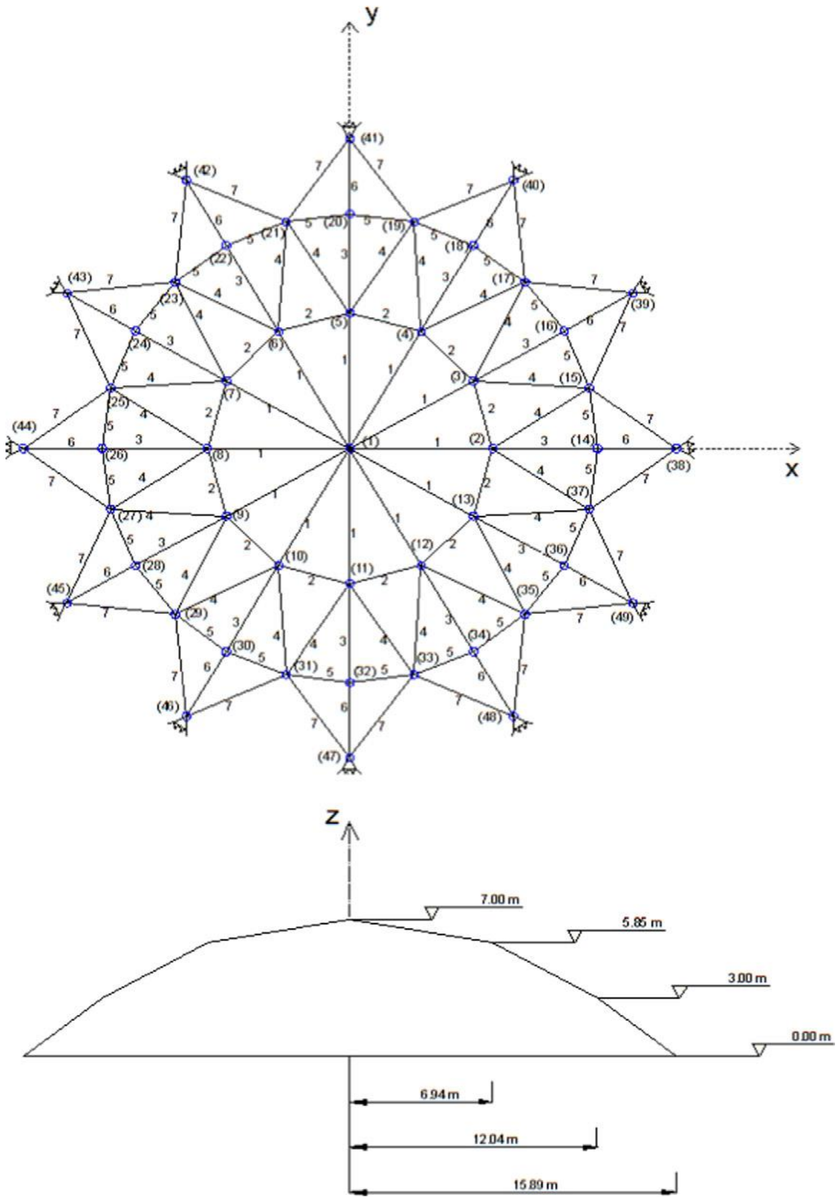


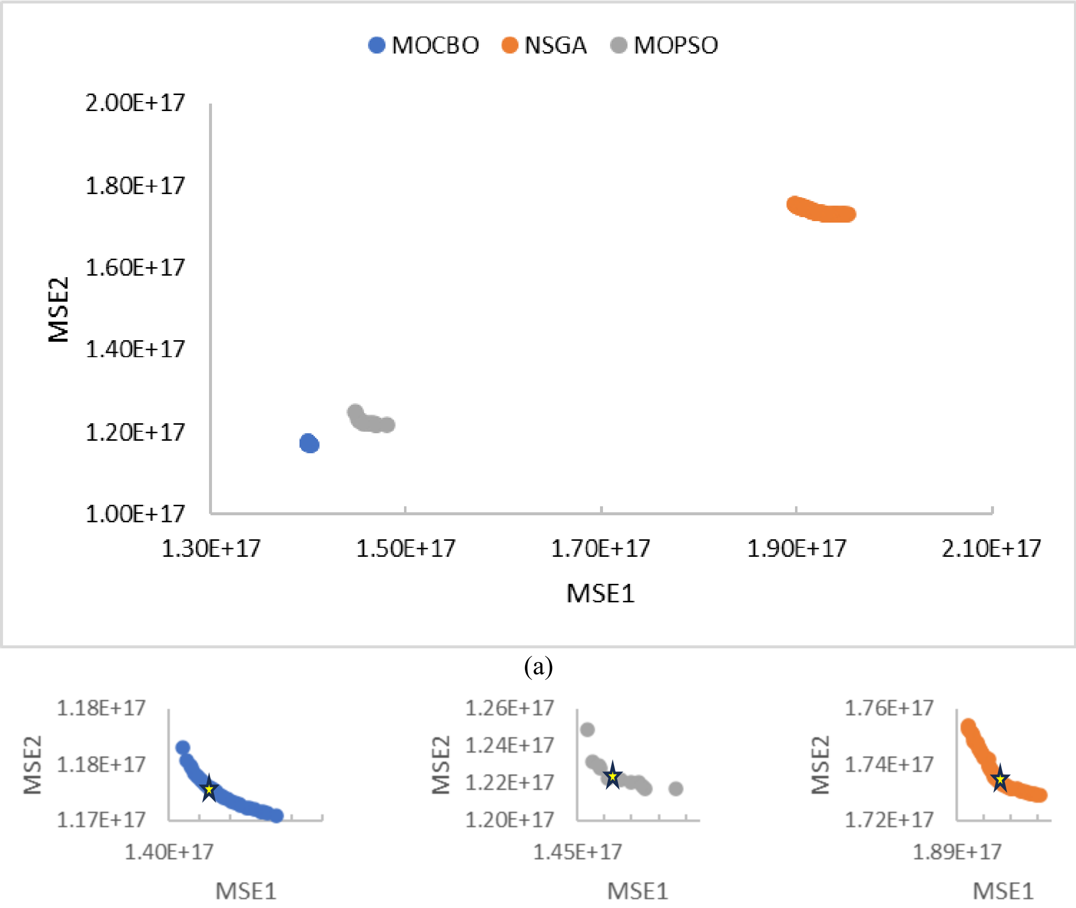

Figure 1: A 120-bar space truss

Table 1: Correlation coefficient of all algorithms for example 1.

| Mode no | GA | PSO | CBO | NSGA-II | MOPSO | MOCBO |
|---|---|---|---|---|---|---|
| **1** | 0.789936 | 0.773575 | 0.842848 | 0.848803 | 0.884482 | 0.889343 |
| **2** | 0.802535 | 0.767703 | 0.820322 | 0.819774 | 0.849033 | 0.852276 |
| **3** | 0.680541 | 0.669648 | 0.631531 | 0.84566 | 0.87359 | 0.879968 |
| **4** | 0.582344 | 0.593871 | 0.605431 | 0.853703 | 0.879261 | 0.882241 |
| **5** | 0.71725 | 0.672688 | 0.737462 | 0.876215 | 0.911193 | 0.916815 |

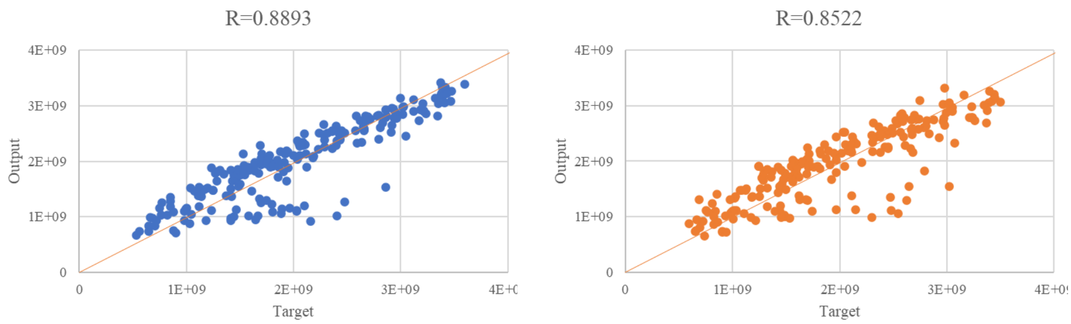


[ DOI: 10.22068/ijoce.2026.16.3.680 ]

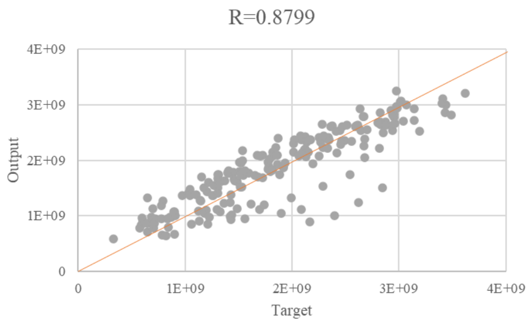



(b) (c) (d)

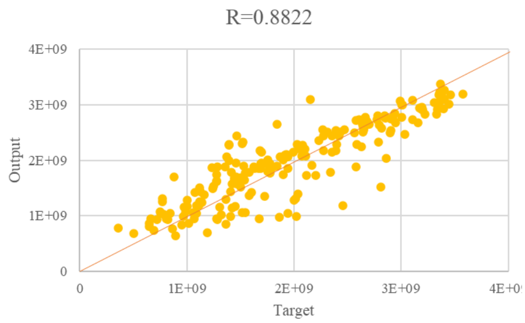
Figure 2: The obtained Pareto Front of MO algorithms for the first example in the first mode: (a) all algorithms, (b) MOCBO, (c) MOPSO, (d) NSGA-II.



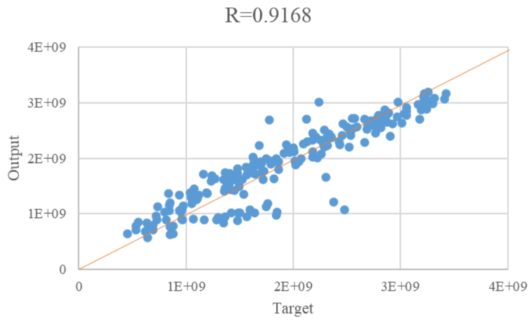

(a) (b)




(c)




(d)




(e)

Figure 3: The observed and predicted values of strain energy for MOCBO algorithm of the first example: a) the first mode, b) the second mode, c) the third mode, d) the forth mode, e) the fifth mode.

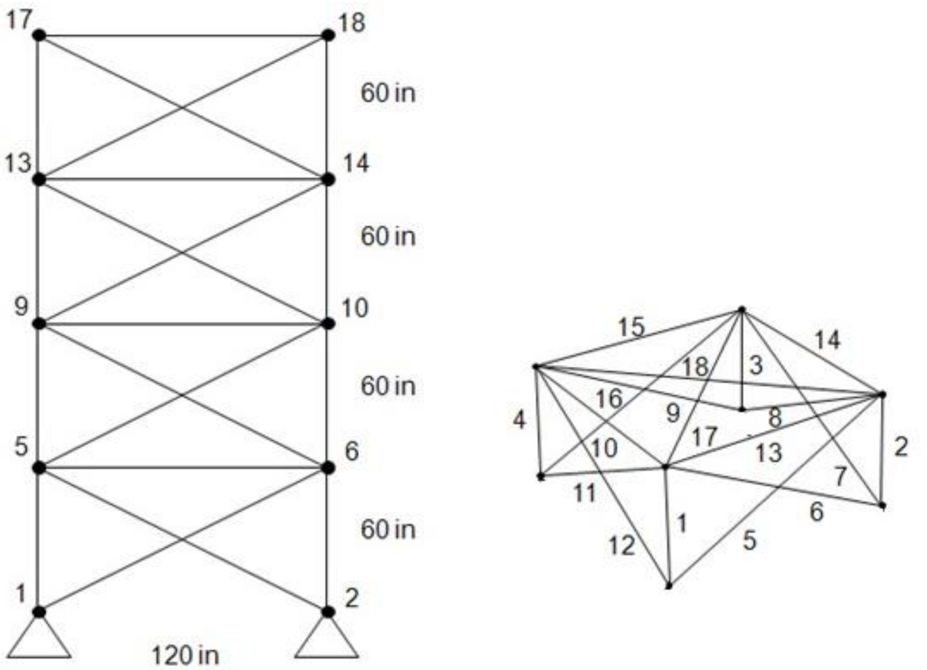
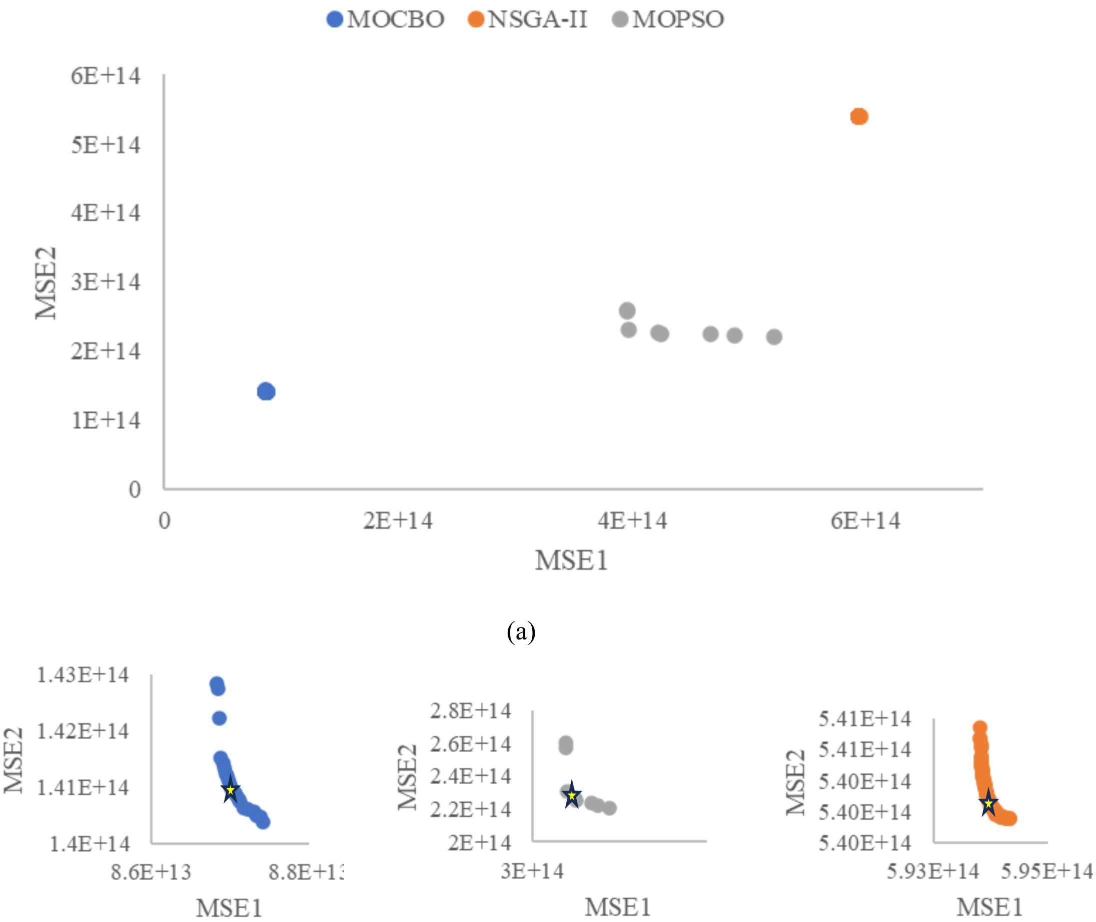



Figure 4: A 72-bar truss; lateral view, top view, and a typical story of the structure are shown.

Figure 5 shows the trade-off between the objective functions using three multi-objective optimization methods. Since the PF sets have more than one solution, the best solution is chosen using the Knee point method described in Ref. [9]. The best solutions found using MO algorithms are also displayed in Figure 2 with a star symbol. The PF set obtained through MOCBO is dominated by those obtained through MOPSO and NSGA-II, as can be observed.




(a)

(b) (c) (d)

Figure 5: The obtained Pareto Front of MO algorithms for the second example in the first mode: (a) all algorithms, (b) MOCBO, (c) MOPSO, (d) NSGA-II.

Table 2 displays the best solutions found in this example using SO and MO algorithms. Figure 6 also illustrates the most accurate predictions of the modal strain energy in different modes using MOCBO algorithm. By comparison, one can see that the predicted modal strain energy using MOCBO is closer to the exact value compared to the outcome of the NSGA-II and MOPSO.



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

Table 2: Correlation coefficient of all the algorithms for example 2.

| Mode no | GA | PSO | CBO | NSGA-II | MOPSO | MOCBO |
|---|---|---|---|---|---|---|
| **1** | 0.898045 | 0.513378 | 0.018496 | 0.442718 | 0.769716 | 0.919279 |
| **2** | 0.880970 | 0.515339 | -0.00640 | 0.441149 | 0.647304 | 0.942903 |
| **3** | 0.918949 | 0.539866 | -0.082840 | 0.461398 | 0.558441 | 0.945489 |
| **4** | 0.857687 | 0.469023 | 0.061728 | 0.405032 | 0.785249 | 0.895946 |
| **5** | 0.542502 | 0.569427 | 0.090920 | 0.550366 | 0.583162 | 0.684312 |

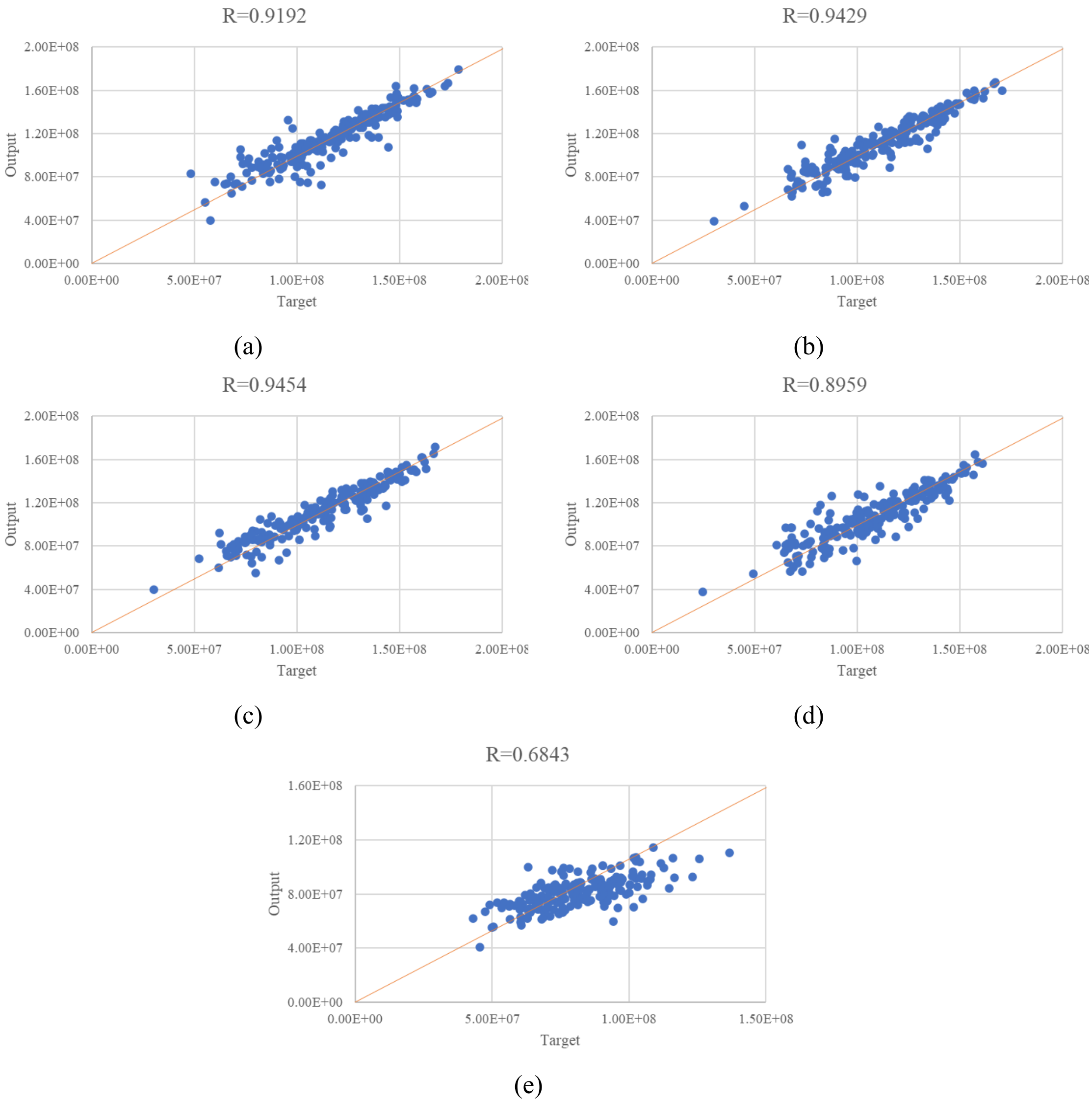



Figure 6: The observed and predicted values of strain energy for MOCBO algorithm of the second example: a) the first mode, b) the second mode, c) the third mode, d) the forth mode, e) the fifth mode.

## 5. CONCLUDING REMARK

This research introduces three multi-objective meta-heuristic algorithms: MOCBO, MOPSO, and NSGA-II, aimed at predicting strain energy in truss structures. Objective functions are first established to determine the optimal weights and biases of a neural network using precise data from the structural analysis. Then, the MO algorithms are used to determine the variables by optimizing the cost functions.

The efficiency of the proposed algorithms is investigated with two spatial truss structures. The results of these examples are analyzed for the strain energy from the first to fifth mode shapes. The obtained results from the numerical studies indicate that the MO algorithms are viable for the problem of neural network training in the strain energy of truss structures.

## REFERENCES

1. Zepeng C, Qitian L, Chudong P. Structural damage detection based on modal strain energy assurance criterion using adaptive region shrinkage assisted IGOA. *Structures*. 2023;**58**:105458.
2. Kaveh A. *Applications of Artificial Neural Networks and Machine Learning in Civil Engineering*. Springer; 2024.
3. Tapeh ATG, Naser MZ. Artificial intelligence, machine learning, and deep learning in structural engineering: a scientometrics review of trends and best practices. *Arch Comput Methods Eng*. 2023;**30**:115–59.
4. Wang C, Chan TM. Machine learning (ML) based models for predicting the ultimate strength of rectangular concrete-filled steel tube (CFST) columns under eccentric loading. *Eng Struct*. 2023;**276**:115392.
5. Kaveh A, Eskandari A, Movasat M. Buckling resistance prediction of high-strength steel columns using metaheuristic-trained artificial neural networks. *Structures*. 2023;**56**:104853.
6. Ledesma-Orozco E, Galvis-Chacón JC, Rodríguez-Sánchez AE. Neural networks modeling of strain energy density and Tsai-Wu index in laminated composites. *J Compos Mater*. 2024;**58**:1217–32.
7. Goichon A, Ghienne M, Tschannen V, Peyret N. Temporal hierarchical modal neural network for structural strain prediction. *Comput Mech*. 2026;**77**:431–49.
8. Kaveh A, Eskandari A, Piri M. Metaheuristic-optimized machine learning for mechanical property prediction in eco-friendly rubberized concrete. *Period Polytech Civ Eng*. 2025;**69**:1124–48.
9. Deb K, Gupta S. Understanding knee points in bicriteria problems and their implications as preferred solution principles. *Eng Optim*. 2011;**43**:1175–1204 → 1175–204.
10. Kaveh A, Mahdavi VR. *Colliding Bodies Optimization; Extensions and Applications*. Springer; 2015.



[ DOI: 10.22068/ijoce.2026.16.3.680 ]

11. Kaveh A, Mahdavi VR. Multi-objective colliding bodies optimization algorithm for design of trusses. *J Comput Des Eng*. 2019;**6**:49–59.
12. Coello Coello CA, Lechuga MS. MOPSO: a proposal for multiple objective particle swarm optimization. In: *Proc 2002 Congr Evol Comput (CEC'02)*. 2002;**2**:1051–56.
13. Deb K. Multi objective genetic algorithms: problem difficulties and construction of test problem. *Evol Comput*. 1999;**7**:205–30.
14. Soh CK, Yang J. Fuzzy controlled genetic algorithm search for shape optimization. *J Comput Civ Eng*. 1996;**10**:143–50.
15. Sedaghati R. Benchmark case studies in structural design optimization using the force method. *Int J Solids Struct*. 2006;**42**:5848–5871 → 5848–71.
16. Kaveh A, Talatahari S. Charged system search for optimum grillage system design using the LRFD-AISC code. *J Constr Steel Res*. 2010;**66**:767–71.



[ DOI: 10.22068/ijoce.2026.16.3.680 ]