



## Heterogeneous Fleet Routing Problem Using Fuzzy Clustering Method by Considering Customer Demand Uncertainty

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### ABSTRACT

Fleet routing is one of the basic solutions to meet the good demand of customers in which decisions are made based on the limitations of product supply warehouses, time limits for sending orders, variety of products and the capacity of fleet vehicles. Although valuable efforts have been made so far in modeling and solving the fleet routing problem, there is still a need for new solutions to further make the model more realistic. In most research, the goal is to reach the shortest distance to supply the desired products. Time window restrictions are also applied with the aim of reducing product delivery time. In this paper, issues such as customers' need for multiple products, limited warehouses in terms of the type and number of products that can be offered, and also the uncertainty about handling a customer's request or the possibility of canceling a customer order are considered. We used the random model method to deal with the uncertainty of customer demand. A fuzzy clustering method was also proposed for customer grouping. The final model is an integer linear optimization model that is solved with the powerful tools of Mosek and Yalmip. Based on the simulation results, it was identified to what extent possible and accidental changes in customer behavior could affect shipping costs. It was also determined based on these results that the effective parameters in product distribution, such as vehicle speed, can be effective in the face of uncertainty in customer demand.

## 1. Introduction

The problem of fleet routing is one of the methods of transferring goods from warehouses to customers, which has been the focus of researchers in recent years. The model of this problem is constantly changing in order to adapt

to new technologies. Getting closer to reality is one of the main goals of the recent research.

Due to the extensive efforts made in fleet routing modeling, various articles have been presented in recent years. The reports recorded from 2009 to 2015 show that in more than 90%

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of the researches, the limitation of the capacity of cars is considered. Also, in more than 37% of the time window issue, and in more than 18% of researches, the issue of returning goods has been raised. The contribution of issues such as the presence of several warehouses, consideration of several time frames, right of priority, delivery of private orders is mentioned in about 5-10%. Also, the share of assumptions such as uncertainty of demand, non-constant speed of cars, dynamic demand and uncertainty of delivery time has reached less than 2% of researches [1]. This low share shows the newness of the topics and the desire of the researchers to achieve the real model of the routing problem. By reviewing articles from 2011 to 2020, references should be used [1-5]. It was found that the objective function of the routing problem includes minimization of the number of cars (70%), minimization of the total distance (82%), minimization of time, total delivery (27%), minimization of station development (18%), minimization of the energy consumption (13%) and minimization of other operating costs (31%) have been modeled as single or multi-objectives [2]. The classification of fleet routing models and its solution algorithms are also among the things mentioned in [3]. The problem of routing with a limited capacity vehicle, routing with a time window, routing with a separate peak, dynamic routing, routing with dynamic demand, routing with simultaneous delivery and pickup, open routing, green routing, etc. are among the most common of these methods. Also, problem solving methods are divided into two categories: exact and meta-heuristic methods, which will not be discussed in detail here [3]. There may be several uncertainty parameters in the routing problem, the most important of which are the uncertainty of the delivery time, vehicle speed, demand amount, cost of goods delivery, traffic conditions, etc., the role of each of them in the problem Green routing is reviewed in [4]. A variety of methods for solving the routing problem have also been presented with details in [5], which include population-based methods (evolutionary competition and particle intelligence) and methods based on a single answer (such as local search methods, etc.).

In recent years, we have witnessed the emergence of various methods for solving the routing problem, the latest research in this field is related to the use of pop music method [6] and deep learning [7] to solve the routing problem with limited capacity, the hybrid congestion method. Particles for clustered routing [8], row and column generation method for routing with a time window [9], and tabu search for time-dependent routing [10] pointed out that most of them succeeded in making a compromise between the accuracy of the answers. and the execution time are optimized. The problem of reducing the calculation time or increasing its accuracy is not raised as a clear advantage of our proposed model, so the details of how it works or the execution time of other researches will not be discussed much. The method used in this article is based on the root and stem method, which will be solved using the combination of two toolkits, Musk and Yalmip. Therefore, we do not claim a new innovation in the field of solving the routing problem. But in order to reduce the optimization time, fuzzy clustering method has been used in this article to make the routing operation faster by reducing the search space of the problem. It should be noted that the number of clusters can have a great impact on the solution time. As the number of clusters increases, the population of each cluster decreases and the routing time of each car decreases, but the increase in the number of cars increases the calculation time. If the number of clusters is reduced, then the search space for each car will increase and the time to get the answers will be longer. Therefore, determining the amount of clusters is very important and necessary.

In the field of improving the routing model and bringing it closer to the real world, valuable measures have been taken. The first problem of the traditional model is the issue of not considering a situation where a car may not be able to arrive at the customer's location within a predetermined time frame. In this case, the traditional model diverges and practically no decision will be made in this situation. In order to avoid this situation or similar cases, considering the penalty for the delay in serving the customer can lead to preventing the divergence of the problem. For the period of

time when service is delayed, the fleet must pay a fee to the customers. Therefore, the priority of place is no longer mentioned and the priority of time is also mentioned. The second problem of the traditional model is the constant issue of assuming the time traveled by each route for all cars. In other words, all traditional model cars have the same constant speed. There is no difference between the speed of light and heavy vehicles. Each of these cars has  $t_{ij}$  equal to other cars. But in practice this is not the case and today we see sending express packages in the market. Therefore, providing express services is also included in this article. The travel time of express vehicles is half that of normal vehicles. The third issue is the participation of customers in the order delivery process. Customers can weigh between their level of comfort and the amount of paying for their courier. The more convenience they want, the sooner the courier will reach them, and in return, the cost of their courier will increase. On the other hand, if they tend to delay more, the amount of their courier cost will decrease. This type of partnership provides the basis for more interactions with customers. The fourth issue is the discussion of the Internet of Things, which provides the infrastructure for the implementation of people's participation. In other words, customers send their weights to the central decision-making unit through the applications they have, and the warehouses also send the data related to the type of goods and their service capacity to the unit through their user software. They send to the decision-making center. Then, through this center, the way of service is determined and the result is communicated from the central unit to each of the warehouses. Despite this knowledge in the network, it is expected that the service process will be modernized and closer to the real world. The fifth problem is to consider a hypothesis for customers' behavior, in which customers usually follow a normal distribution and do not think alike. This normal distribution is closer to the real world, but does not represent the worst cases in the transportation network. The sixth issue is the issue of offline (a day ahead) and online decision making. It is possible that the customer may request to cancel or increase the items of his order despite sending his order from

the previous day. Therefore, traditional offline methods (once for the whole day) will no longer work and online decision-making methods are needed. In this case, using the control method of the predictive model can be very effective and provide the conditions for decision-making in any time frame of optimization.

The contribution of this paper compared to the traditional model considering the window are:

- In order to avoid the situation, where a car may not be able to arrive at the customer's location within a predetermined time frame, we considered the penalty for the delay in serving the customer to prevent the divergence of the problem.
- The travel time of different car types is considered as a dynamic parameter so that the travel time of express vehicles is half that of normal vehicles
- interactions with customers is provided in this paper for the first time, so that the customers can weigh between their level of comfort and the amount of paying for their courier
- the capability of communicating through Internet of Things technology is provided in the model
- To mimic the customers' behavior according to the real world, we used the normal distribution for the customer's request
- Decision-making at any time frame of the optimization process is provided so that the customers are able to change their orders at any time

Other parts of this article will be presented as follows. In part 2, the traditional model and the proposed model are introduced. Section 3 is dedicated to the problem solving method and its implementation flowchart. Section 4 is related to the results of simulating the problem in MATLAB software and examining different case studies. Finally, the conclusions will be presented in section 5.

## 2. The proposed method

In this section, the proposed method will be presented based on the new modeling and then the preliminary solution method for its optimal solution will be described. The first subsection, i.e. 2.1, addresses the traditional model considering the time window while subsections 2.2 and 2.3 provide the formulations to complete and make the basic model more real by considering path-dependent physical limitation, capacity-dependent physical limitation, time window limitations and specially the uncertainty.

### 2.1. Traditional model considering the time window

Fleet routing problem includes various binary and real variables. We define the variable  $X_{ijktp}$  as the symbol of the passage of the  $k$ -th car (belonging to warehouse  $p$  and carrying product  $t$ ) from node  $i$  to node  $j$ , and its value of one means passing and zero means not passing. Also, we consider the variable  $Y_{ij}$  as a real variable representing the total capacity of all vehicles passing from node  $i$  to node  $j$ . We define the  $W_{ikpt}$  variable as the time when vehicle  $k$  (belonging to warehouse  $p$  and containing product  $t$ ) passes node  $i$ . These three variables are the basis of fleet modeling relationships.

The goal of routing is usually considered to minimize the distance traveled by cars, which means reducing the amount of pollutants and using the maximum capacity of cars. The mathematical representation of this objective function is:

$$\min \text{obj} = \sum_p \sum_t \sum_k \sum_j \sum_i D_{ij} x_{ijktp} \quad (1)$$

In this regard,  $D_{ij}$  is a known quantity that represents the distance between two nodes  $i$  and  $j$  from each other.

The management of the transportation fleet is subject to special regulations that form the structure of the border restrictions of this issue.

#### 2.1.1. Path-dependent physical limitation

It is necessary to provide each customer's goods by the transportation organization only once.

$$\sum_p \sum_k \sum_i x_{ijktp} = \delta_{jt}; \forall t, j \quad (2)$$

In this regard,  $\delta_{jt}$  is a binary variable? If node  $j$  has a request for product type  $t$ , this parameter is equal to one, and in other cases, this value is equal to zero.

It is not possible to return to the previous node (or round) for any vehicle.

$$x_{ijktp} + x_{jiktp} \leq 1, \forall i, j, k, t \quad (3)$$

The origin and destination of a route can never be the same.

$$x_{iiktp} = 0, \forall i, k, t, p \quad (4)$$

The car of each warehouse belongs to the same warehouse and cannot go to other warehouses or come from another warehouse.

$$\sum_j x_{p_m j k t p_n} + \sum_i x_{i p_m k t p_n} = 0 \quad (5)$$

Every exit from the car must lead to the entry of the same car into the parking lot.

$$\sum_i \sum_{j \in p} x_{ijktp} = \sum_{i \in p} \sum_j x_{ijktp} \quad \forall k, t, p \quad (6)$$

#### 2.1.2. Capacity-dependent physical limitation

Regarding the capacity, there are several limitations in the problem: one is the limitation of the vehicle capacity, the limitation of the number of vehicles and the limitation of the goods distribution warehouse. In many cases, for the sake of simplicity, it is assumed that the warehouse does not have a capacity limit, but in practice, this is not the case. Limiting the number of cars is also effective in the obtained results.

If each car has capacity  $Q_k$ . The amount of cargo delivered from warehouse  $p$  must be less than its own capacity. So we will have:

$$\sum_j \sum_t y_{pjkt} \leq Q_k; \forall p, k \quad (7)$$

In this regard,  $y_{pjkt}$  is the volume (or capacity) of goods of type  $t$  passing from node  $i$  to node  $j$  by car  $k$ , which is related to warehouse  $p$ . The relationship between  $x_{ijkt}$  and  $y_{ijkt}$  is shown in the following relation.

$$y_{ijkt} = C_t x_{ijkt}; \forall i, j, k, t, p \quad (8)$$

In this regard,  $C_t$  represents the capacity (or volume) occupied by the product of type  $t$ . After the car reaches each customer, the amount of cargo unloaded from the car is equal to the amount needed by the customer. So we have:

$$\sum_p \sum_k \sum_i y_{ijkt} - \sum_p \sum_k \sum_i y_{jiktp} = d_{j,t}; \forall j, t \quad (9)$$

Similar relations can be expressed about the capacity of each warehouse:

$$\sum_j \sum_k \sum_t y_{pjkt} \leq Cap_p; \forall p \quad (10)$$

In this regard,  $Cap_p$  is the capacity of storage  $p$ . The above relationship states that the volume of goods sent from the warehouse  $p$  (the total load of all vehicles carrying the types of goods in the warehouse  $p$ ) must be less than the warehouse capacity.

### 2.1.3. Time window limitations

The process of moving goods from the warehouse to the customer's place always requires a certain amount of time. The delay in sending the goods always leads to the inconvenience and complaints of the customers. Therefore, it is necessary to always include a time limit in the problem in order to obtain far more logical answers. If the moments  $s_{ikt}$  and  $s_{jkt}$  represent the arrival time of car  $k$  carrying goods  $t$  from nodes  $i$  and  $j$ , then by taking into account the time traveled by the car on the transportation route of these two nodes  $t_{ij}$ , it can be written:

$$x_{ijkt}(s_{ikt} + t_{ij} - s_{jkt}) \leq 0; \forall k, t, i, j \quad (11)$$

In other words, this relationship states that if car  $k$  moves from node  $i$  at time  $s_{ikt}$ , it can reach node  $j$  after passing the minimum time  $t_{ij}$ . Another point is that the arrival time of cars to each node should not be outside the expected range of customers. So we have:

$$T_{ait} \leq s_{ikt} \leq T_{bit}; \forall k, t, i, j \quad (12)$$

In this regard,  $[T_{ait}, T_{bit}]$  is the expected interval of receiving product  $t$  by customer  $i$ .

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$$x_{ijkt}(s_{ikt} + t_{ij} - s_{jkt}) \leq 0; \forall k, t, i, j \quad (13)$$

In other words, this relationship states that if car  $k$  moves from node  $i$  at time  $s_{ikt}$ , it can reach node  $j$  after passing the minimum time  $t_{ij}$ . Another point is that the arrival time of cars to each node should not be outside the expected range of customers. So we have:

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### 2.2. Uncertainty

The uncertainty considered in this article is related to the quantity of goods requested by subscribers and the proposed time frame for receiving their orders. In other words, uncertainty parameters are defined as the following relationships:

$$\tilde{d}_{j,t} = (1 + \varepsilon_{j,t})d_{j,t}, \quad \varepsilon_{j,t} \in Normal(0,1) \quad (15)$$

$$\tilde{T}_{ait} = 0.5(T_{ait} + T_{bit}) - 0.5(1 + \varepsilon_{j,t})(T_{bit} - T_{ait}), \quad (16)$$

$$\tilde{T}_{bit} = 0.5(T_{ait} + T_{bit}) + 0.5(1 + \varepsilon_{j,t})(T_{bit} - T_{ait}), \quad \varepsilon_{j,t} \in Normal(0,1) \quad (17)$$

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In these relations  $\tilde{d}_{j,t}$ ,  $\tilde{T}_{ait}$  and  $\tilde{T}_{bit}$  are respectively the amount of indeterminate requested load, the beginning of the order receiving period and the end of the order receiving period desired by the customer. Also,  $-1 \leq \varepsilon_{j,t} \leq 1$  is a random variable that follows the normal distribution function  $N(\mu = 0, \sigma^2 = 1)$ . By changing from negative one to one the value of customer demand in the interval  $0 \leq \tilde{d}_{j,t} = (1 + \varepsilon_{j,t})d_{j,t} \leq 2d_{j,t}$ , the length of the customer's time interval. In the interval  $0 \leq \tilde{T}_{bit} - \tilde{T}_{ait} = (1 + \varepsilon_{j,t})(T_{bit} - T_{ait}) \leq 2(T_{bit} - T_{ait})$  will change. Note that the center of the time interval requested by the customer has not changed  $\frac{(\tilde{T}_{bit} + \tilde{T}_{ait})}{2} = \frac{(T_{bit} + T_{ait})}{2}$  and only the length of the time interval has become smaller or larger.

### 2.3. The Proposed stochastic model

In order to deal with the uncertainty, the stochastic model is presented. Also, in order to prevent the problem from diverging due to the time limit proposed by the customers, a penalty has been considered for delay or excessive haste. In other words, if the delivery time of a product is delayed or accelerated by  $E_i$  compared to the customer's proposed interval, in this case, a total penalty equal to  $\sum_i C^{penalty} E_i$  will be added to the objective function. Therefore, the fleet routing stochastic model is defined as follows:

$$\min \sum_{sen} (obj + C^{penalty} \sum_i E_i) \quad (18)$$

s. t Equation 1-8 and 10-11,  $\forall sen$

$$\sum_p \sum_k \sum_i y_{ijktp} - \sum_p \sum_k \sum_i y_{jiktp} = \tilde{d}_{j,t}; \forall sen$$

$$\tilde{T}_{ait} - E_i \leq s_{ikt} \leq \tilde{T}_{bit} + E_i; \forall k, t, i, j, \forall sen$$

In other words, the objective function of the random model is the sum of the objective function obtained in each scenario. Also, restrictions 1 to 8 and 10 to 11 are independent of the scenario. However, the amount of the customer's desired demand and the time frame of his receipt will change based on the scenario, and if it is still delayed or accelerated compared to the customer's proposed time frame, then a penalty

will be considered. The worst worst-case performance of the model will be realized when all the variables of the problem are constrained in terms of boundary and also the value type which leads the problem to be as one the mixed integer programming problem category which its time and spatial complexity would be in the non-polynomial order.

### 3. The proposed Optimization algorithm

The process of solving the optimization problem is shown in the figure below. As it is clear in this figure, after starting the program and reading the problem information, the program is executed for the first scenario, and the results are saved. Then, until the scenarios are completed, this optimization is also implemented for other scenarios and its results are saved. Finally, the obtained results are integrated and the results are presented randomly. It should be noted that in each new scenario, all the information is similar to other scenarios, with the difference that the amount of demand of each customer and the proposed time of receiving their goods will change. Since changes of these parameters are not certain, therefore, in different scenarios, changes are applied to these parameters according to the normal distribution function and the result of its optimization is obtained. This process is implemented for a number of scenarios (10 scenarios in this article) and its results are saved for statistical analysis.

In order to maintain the comprehensiveness of the problem solving, in the above figure, there is not much explanation about the details of how to implement the optimization for each of the scenarios. How to perform optimization for each scenario plays an important role during program execution. If all the customers, goods and warehouses are to be used in the optimization at the same time, then the duration of the program will be greatly increased and it will not be possible to access the answers in a reasonable time. One of the ways to reduce the calculation time of the program is to assign potential customers to each warehouse. There are two criteria for choosing a warehouse for each customer. One is the availability of that product in the warehouse and the second is the distance between the customer and the warehouse. In a

normal situation, any warehouse that has the shortest distance to the customer must supply the product to that customer, but if that warehouse does not have the product desired by the customer, the next closest warehouse that has the stock of the product must supply the stock. Take over the customer. In this way, the customers of each warehouse are allocated for each product. Because it is possible that each of the goods needed by the customers can be supplied from different warehouses, therefore the optimization of each of the goods will be done separately.

The second issue arises after allocating customers related to each warehouse, and that is the high number of customers of some warehouses. If there are many customers of some warehouses, it will still take time to solve the problem. Therefore, it is necessary to simplify again in such cases. At this stage, clustering is done based on fuzzy logic, in which customers who are close to each other are placed in a cluster. In this article, it is sufficient to divide the customers of each warehouse into two clusters.

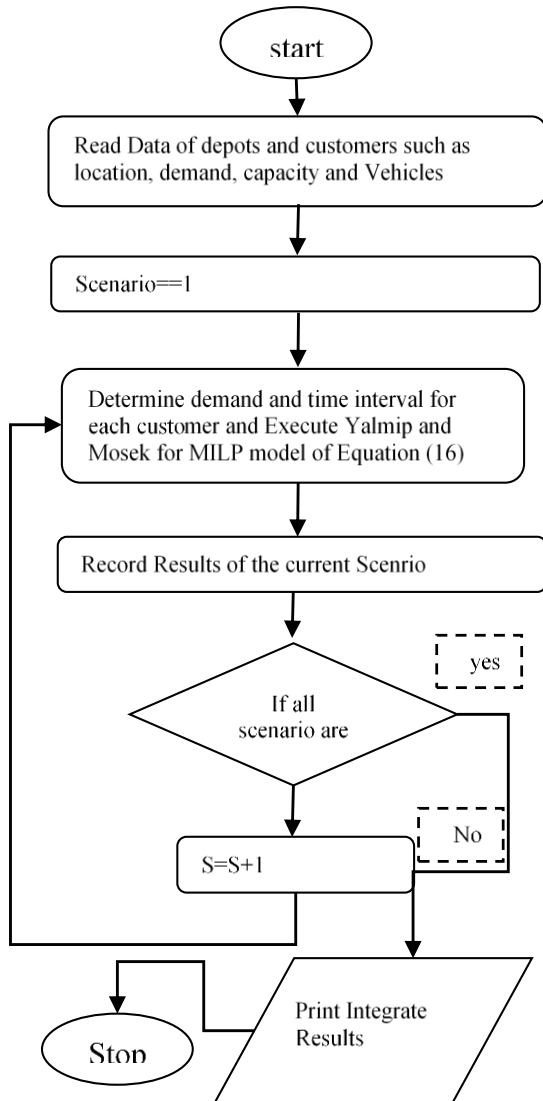
#### 4. Simulation and analysis of results

In this article, the change of a network with 4 warehouses, 48 customers and two types of goods is studied. At first, it is assumed that some warehouses do not have some goods, as shown in Table 1.

**Table 1: The status of goods in warehouses**

	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4
Goods 1	1	1	0	1
Goods 2	1	1	1	0

Information related to the location of each customer and warehouse, information related to the amount of customer demand, basic information related to the time period desired by the customer to receive his goods, etc. are also mentioned in the reference. In order to solve the stochastic model of the problem, the standard normal distribution is included for the variable so that the customer's demand can change from zero to twice his normal demand based on the normal distribution. Also, the length of the customer's expected interval can change from a momentary amount to twice the normal length of the expected interval. In each scenario, a random number is assigned to each of the customers for changing their time period and demand, and the



**Figure 1: The Optimization algorithm Flowchart**

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optimization problem is done for it to determine what behavior should be done in each scenario and the final distance traveled compared to the case without Uncertainty in what state it will be. On average, in this network, about 20 customers are assigned to each of the warehouses, in the meantime, in order to reduce the calculation time, fuzzy neural clustering method has been used in such a way that the number of elements in each cluster does not exceed 10. To solve the problem faster. In other words, with the increase in the number of customers, the number of boundary constraints of the routing problem increases exponentially, which has caused the problem to be solved more slowly. Therefore, there are 10 customers in each cluster and 5 cars in each cluster. The capacity of 200 units is responsible for supplying the loads of that cluster. A hundred scenarios are considered.

The results related to the supply of type 1 and type 2 products are shown in figures 2 and 3. As it is known, graphs A to C are assigned to scenarios 1 to 3 respectively. It is clear that due to the changing needs of customers and their expected time, the goods supply network has been forced to change various routes based on the new priorities of customers so as to reduce its routes as much as possible and pay less fines. When the customers' demand changes randomly, especially the time they offer to receive the goods changes, priority will be given to customers who have the limit of faster supply of their demand.

Therefore, we will see a change in the distance traveled by goods. The shorter the supply time of a product, the higher its supply priority. As the amount of load desired by customers increases, more vehicles or higher capacity will be needed. However, the results will change and the definitive method will no longer meet the needs of customers. Adaptability to such changes will not be possible without the presence of communication or telecommunication infrastructure such as the Internet of Things. It is through the Internet of Things that each of the customers can immediately inform the change of their needs and the transport organization can adapt itself to the needs of the customers in this way.

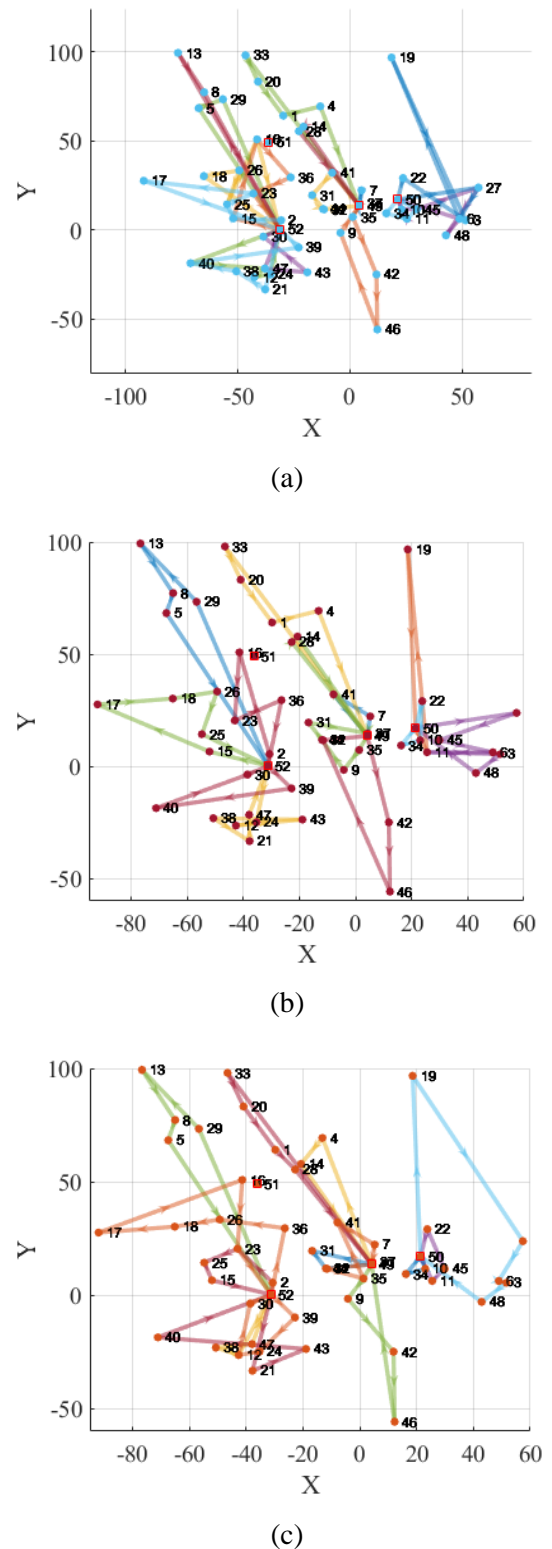


Figure 2: Optimal routing for supply of type 1 goods in a) scenario 1, b) scenario 2 and c) scenario 3



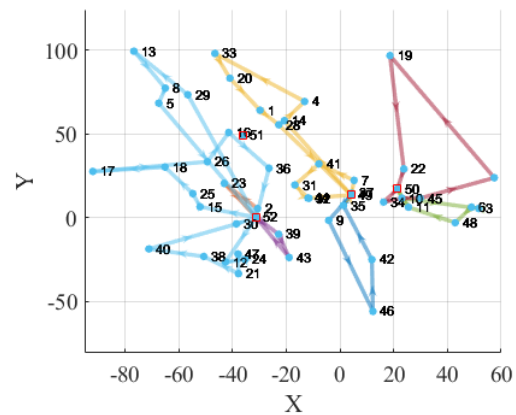
**Table 2: The total distance traveled in each of the scenarios**

Percentage growth of the distance compared to the deterministic model	Product distance 2	Percentage growth of the distance compared to the deterministic model	Product distance 1	Scenario
0	1392	0	1760	No uncertainty
36.505	1900	-2.033	1725	1
0.222	1395	-5.815	1658	2
3.210	1437	0.7610	1774	3
1.224	1409	-20.621	1397	4
5.506	1462	-4.532	1681	5
10.967	1532	-21.240	1386	6
26.42	1760	9.206	1922	7
5.960	1457	-8.467	1611	8
19.64	1666	4.305	1836	9
2.743	1430	1.766	1729	10

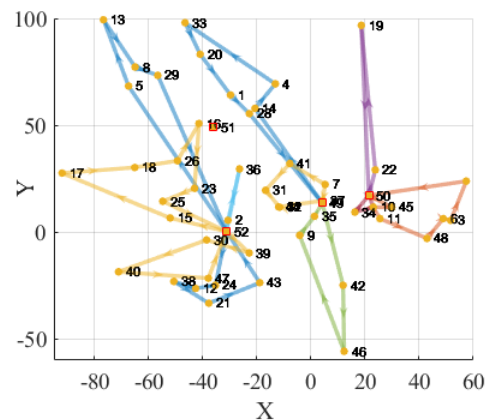
In Table 2, the result of each of the scenarios (the first ten scenarios) and their differences with the deterministic model are compared from the point of view of the total mileage. It should be noted that the more the number of scenarios, the more logical the answers will be. As can be seen, because the selected cars are all normal cars with normal speed, we see an increase in the distance compared to the definite case. If fast cars are used, and it is possible to transport goods faster, the problem will be solved better.

**Table 3: The total distance traveled in each of the scenarios in the case of increasing the speed of the cars by 50%**

Percentage growth of the distance compared to the deterministic model	Product distance 2	Percentage growth of the distance compared to the deterministic model	Product distance 1	Scenario
0	1167	0	1381	No uncertainty
6.102	1238	-1.036	1367	1
-1.683	1147	2.977	1658	2
6.179	1239	24.860	1724	3
9.183	1281	26.726	1750	4
5.119	1226	4.861	1448	5
9.348	1276	8.543	1499	6
7.404	1253	7.873	1490	7
-0.641	1159	4.596	1444	8
3.329	1205	6.053	1464	9
0.997	1178	2.146	1411	10

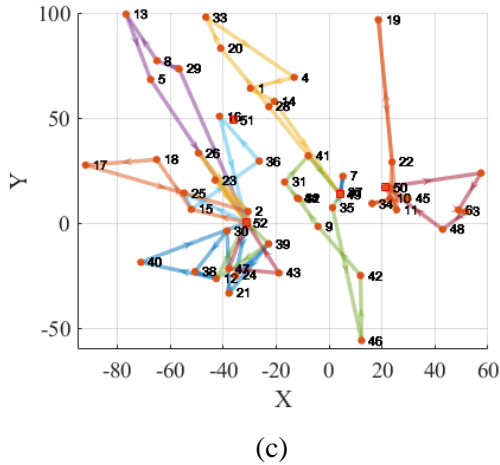


(a)



(b)

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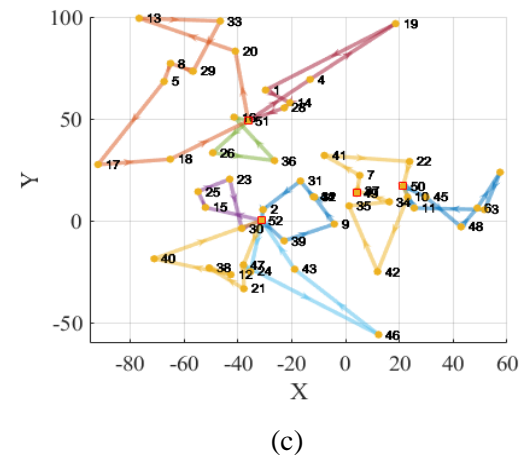
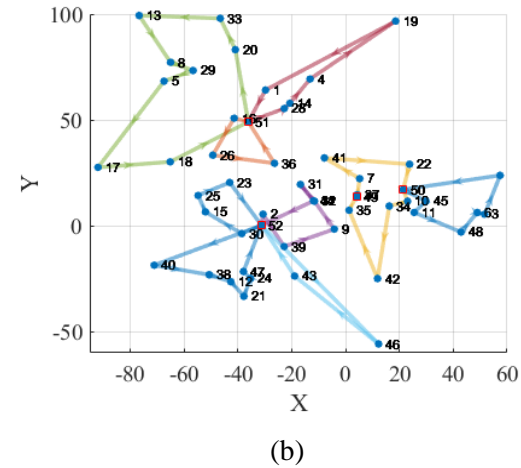
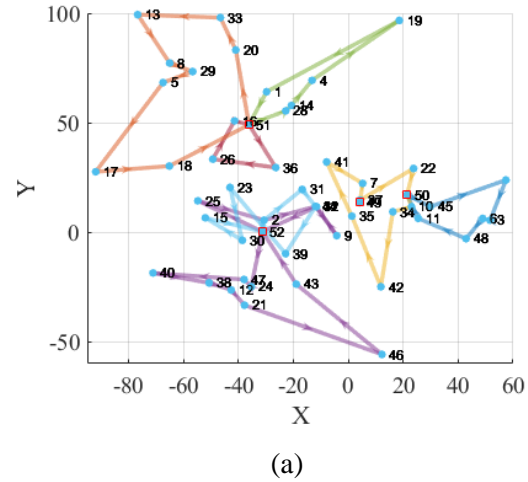
**Figure 3: Optimal routing for supply of type 1 goods in a) scenario 1, b) scenario 2 and c) scenario 3**

Table 3 and Figures 3 and 4 show the result of doubling the speed of some vehicles in the results of the problem. As it is known, the number of network loops has been reduced and instead one car has covered several nodes. In other words, if the speed of the car increases, it can meet the desired need with a smaller number of cars.

### 5. Conclusion

In this article, the fleet routing problem was modeled by considering the uncertainty of subscribers' behavior in their orders. Types of storage capacity restrictions, time window restrictions, vehicle capacity, distance traveled, etc. were included in the problem. And the effect of changing the speed of cars was investigated. A stochastic model to deal with uncertainty is stated and its result was analyzed for a sample network. The results show that depending on the type of random event created for network customers (in order to reduce or increase their demand), the final distance traveled to supply the load is less or more than the determined model. Therefore, the distance depends on the behavior of customers. If a load is canceled, no further distance will be traveled to provide it or another load will be provided, however, less distance has been traveled compared to the deterministic model in which cancellation was not considered. Also, if the load increases, we see an increase in the distance traveled to supply that load, and the total

distance will be greater compared to the deterministic model, this shows the intelligence



**Figure 4: Optimum routing for supply of type 2 goods in a) scenario 1, b) scenario 2 and c) scenario 3 by increasing the speed of the car by 50%**

and flexibility of the program in quickly providing load fluctuations, which does not exist in the deterministic model. Therefore, the necessity of using these random methods will be to respond faster to the needs of customers. Another important and significant point is that the participation of customers in the time of supplying their cargo also plays a very important role in the amount of distance traveled and the way of routing. Customers who were not interested in participating and had an urgent request to meet their needs have been given the first priority to respond, and the flexibility of other customers has led to the car supplying other loads by respecting the right of priority. The results show that considering the penalty for late delivery to the customer, all customers have been answered within their desired time frame and the customer has not been serviced outside of the suggested time. In a way, this fine is considered as an incentive for more customer participation and increasing the quality of service. Therefore, the proposed method is closer to the real method.

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