Fuzzy Vehicle Dynamic Control for a Three-Wheeled Vehicle Using Tilt Mechanism

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Abstract

Nowadays, the use of small vehicles is spreading among urban areas and one sort of these vehicles are three-wheeled vehicles (TWVs) which can be competitive with four-wheeled urban vehicles (FWVs) in aspects such as smallness, simple manufacturing, and low tire rolling resistance, fuel consumption and so on. The most critical instability associated with TWVs is the roll over. In this paper a tilt control mechanism has been modeled which can reduce the danger of roll over by leaning the vehicle towards the turning center in order to decrease the amount of lateral load transfer (LLT), and by doing so, system combines the dynamical abilities of a passenger car with a motorcycle. A 3 degree of freedom vehicle model is simulated at constant speed in MATLAB-Simulink environment and a fuzzy algorithm is developed to control such a non-linear system with appropriate tilting torque. Results are interpreted in presence and absence of controller with different longitudinal speeds and steering inputs; the results are also compared to behavior of a similar FWV and this is concluded that the tilt control system could countervail deficiencies of the TWV compared to the FWV.

Keywords*: Three-Wheeled Vehicle, Tilt Mechanism, Roll Over, Fuzzy Control, Vehicle Dynamics, Simulation*

1. Introduction

Today, small vehicles play a significant role in our daily life which causes the small city cars to be very popular among people especially in crowded regions. Low fuel consumption, low emission, small dimensions and affordable prices are some reasons for such popularity [1]. TWVs are also placed in this category and are considered as small urban vehicles which are also used in countries such as China and India as transporter vehicles and sometimes for cargo transfer reasons. A noticeable advantage of these vehicles in comparison to FWVs is their ability to combine powertrain and suspension mechanisms used in motorcycles and passenger cars in order to minimize the complexity of systems; for instance, if the power is to be transferred to the single wheel, the mechanism will be easily designed without any need to differentials; or if the single wheel is located at the front, the steering mechanism could be a direct steering mechanism or any system much simpler than the conventional FWV. Generally, these kinds of modifications could also cause in total weight reduction and subsequently better fuel economy.

Instabilities associated with TWVs can be categorized into two types: lateral issues arising from the loss of lateral force generation ability by the single wheeled axle; and more importantly, the deficiency of the TWVs in withstanding the LLT generated in turns and their high tendency to roll over in turns and decreasing the roll over threshold. This arises from the fact that all LLT generated in the turn should be compensated by the normal tire loads in the two-wheeled axle [2].

A special control system used to counteract the roll over threat of TWVs is the Tilt Control (TC) mechanism. There are two types of TC: 1) Direct Tilt Control (DTC), 2) Steering Tilt Control (STC) [3].

DTC: In this system, actuators are located between the sprung and un sprung masses (or integrated with suspension such as the Mercedes F300). A lateral acceleration sensor is used to obtain the current status of the vehicle and the control system determines the required torque to locate the vehicle in proper tilting position at the moment. The main disadvantage of such systems is the probable time delay and complicated control algorithm. On the other hand, establishing such a system will not require the driver to possess special skills and information about the system functioning [3].

Piyabongkam et al. conducted a research in which a prototype is made and two control approaches are conducted for the case (RHC Controller and a nonlinear controller using feedback linearization) [4].

Nestor Roqueiro qt al. developed a sliding mode direct tilt control for a TWV and results are analyzed [5].

STC: This method is actually the same method that a bike rider uses unconsciously when tilting the bike in maneuvers. Generally speaking, this method is dependent almost to the speed, steering angle and the driver experience. The main advantage of this system is the fast response.

In a research by Pohl and Conrads a fully mechanical single passenger prototype is made in Which the driver tilts the vehicle by pushing two reciprocal pedals which is relying totally on the physical ability and experience of the driver [6].

TC not only compensates the effect of LLT, but also increases the lateral tire forces by generating favorable camber angle in the tires of the vehicle.

In the following sections, a DTC is designed and the simulation is done using MATLAB-Simulink on a simplified TWV model with a single wheel in the front axle.

2. 2. Vehicle Model

3. 2.1. Assumptions

Constant Longitudinal Speed: Vehicle speed is considered constant in order to eliminate the longitudinal motion equation. This assumption is reasonable in most of the maneuvers.

Even Road Surface: The road is considered flat and no imperfection is considered in the analysis. This assumption enables us to focus on the tilting required torque directly.

Single Wheel Location: According to the results of previous work [1] it is concluded that in order to have more understeering behavior which is considered safe and stable compared to over steering, the best location for the single wheel is at the front end of the vehicle.

Ignored Wheel Travel: In calculation of the tire normal loads, it is assumed that the wheel has no significant mass and the tire normal forces are instantly transferred to the suspension and the sprung mass; but since we are looking for a comparison between TWVs and FWVs in presence and absence of TC, this assumption will not impair the simulations.

Roll Center Height: As the design of suspension and tilting mechanism is beyond the scope of this investigation, the roll center height is assumed to be zero making the tilt axis coincident with the vehicle longitudinal axis.

Vehicle parameters are given in Appendix.

2.2. Equations of Motion [1]

SAE coordinate convention is used for obtaining the equations of motion [7].

Lateral Motion:

$$Fy,total=Fy,f+Fy,rl+Fy,rr=Mt(Vy+Vx.\dot{\psi})=Mtay \quad (1)$$

Roll Motion (Uncontrolled Vehicle): $\Sigma Mx=(Fz,rr-Fz,rl).Tr2-Fy,al(hCG-hCR)=Is,xx.\ddot{\varphi} \quad (2)$

Tilt Motion (Controlled Vehicle):

$$M+(hCG-hCR).\sin\varphi-Fy,total(hCG-hCR).\cos\varphi=Is,xx.\ddot{\varphi} \quad (3)$$

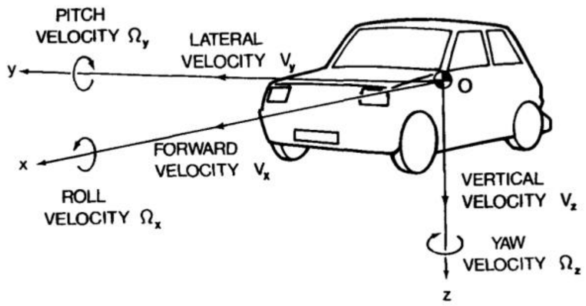


Fig1. Vehicle Coordinate System

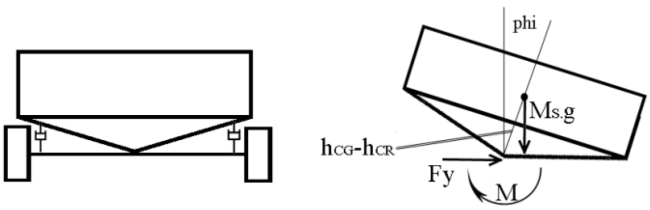


Fig2. Vehicle Tilt Model

Fig3. Mercedes F300 Tilting

Yaw Motion:

$$\Sigma \boldsymbol{M_z} = (\boldsymbol{F_{y,f}}).\boldsymbol{L_f} - (\boldsymbol{F_{y,rr}} + \boldsymbol{F_{y,rl}}).\boldsymbol{L_r} + \boldsymbol{M_{z,total}} = \boldsymbol{I_{zz}}.\ddot{\boldsymbol{\psi}} \quad (4)$$

$$\boldsymbol{M_{z,total}} = \boldsymbol{M_{z,f}} + \boldsymbol{M_{z,rl}} + \boldsymbol{M_{z,rr}} \quad (5)$$

Tire model used in this research is the Magic Formula tire model [8]. Since we are dealing with fuzzy control, there will be no limitations for linear equations throughout the vehicle model, so it has been chosen to use Magic Formula in order to get more realistic results compared to other possible tire models. The details of the tire cornering and moment characteristics can be found in [8] and [2].

Equations of motion for the FWV are much similar to those of TWV and are omitted for the sake of space saving.

3. Tilt Mechanism and Control

3.1. Mechanism

Tilting mechanism used for simulation in this paper is adopted from Mercedes-Benz F300 which utilizes a double-wishbone suspension system manipulated by hydraulic actuators to imply the tilting moment between the sprung and un sprung masses [6]. The only difference is that the effect of springs is not considered in this analysis because of simplicity and the type of analysis performed; in other words, the tilting moment is directly applied between the wheels (suspension arms) and the body.

This special double wishbone configuration is a little different from conventional samples; the upper arm is longer than the lower one due to the technical fact that for better maneuverability and lateral force we need negative camber for the outer wheel and positive camber for the inner wheel in turns, while the vehicle body is leaning towards inside of the turn despite normal cars which tend to roll to the outside of the turn. Thus the suspension geometry and the kinematics are different in this manner. Since the design of suspension system is not in the scope of this paper, we use the results obtained by visual inspection as the kinematic analysis of suspension geometry to determine the camber angles associated with a certain tilting angle as follows:

$$\gamma_f = \varphi \quad (6)$$

$$\gamma_{rr} = 0.375\varphi - 1.25 \quad (7)$$

$$\gamma_{rl} = 0.25\varphi - 1.5 \quad (8)$$

According to the vehicle dynamics equations mentioned before, one can see that the whole system is extremely nonlinear which strongly implies the use of a nonlinear controller to take charge. Among nonlinear control systems, the fuzzy control is a convenient approach if the designer has good understanding of the system's behavior. In this paper, this type of controller is used to get the desired tilt angle by applying the proper torque.

3.2. Control Approach

In the first stage, a Mamdani controller was designed but the results were not satisfactory at all because according to the complexity of the system it is not easily possible to define constant membership functions for the tilting torque such as "High", "Low", etc. Thus a TSK controller is designed using special and while very simple mathematical expressions ($\widetilde{\boldsymbol{M}}$ and $\boldsymbol{M_{normal}}$) which are described later.

3.3 Desired Tilt Angle

At first, we need to define a desired tilt angle for the vehicle in turns. This is done by thinking like the case of a motorcycle in turn; we equal the LLT to zero to ensure having the maximum roll stability

$$\Sigma F_y = 0 \rightarrow F_y = M_t a_y$$
$$\Sigma F_z = 0 \rightarrow F_{z,f} + F_{z,rl} + F_{z,rr} = M_t g$$
$$\begin{aligned}\Sigma M_{CG} = 0 \rightarrow M_t a_y h_{CG} \cos\varphi &= -F_{z,rr}\left(\frac{T_r}{2} - h_{CG}\sin\varphi\right) \\ &+ F_{z,rl}\left(\frac{T_r}{2} + h_{CG}\sin\varphi\right) \\ &+ F_{z,f} h_{CG}\sin\varphi = M_t g h_{CG}\sin\varphi \\ &\rightarrow \tan\varphi = \frac{a_y}{g} \rightarrow\end{aligned}$$
$$\varphi = \tan^{-1}\left(\frac{a_y}{g}\right) \tag{9}$$

3.4. Controller Design

Tilt angle error and its rate of change are considered as controller inputs which need to be fuzzified. Maximum tilt angle is considered 45 degrees and the following membership functions are developed to fuzzify the controller inputs,

Now we define a new variable called $\widetilde{\boldsymbol{M}}$ and it is defined as follows:

Assuming a constant angular acceleration for the vehicle body in tilting, applying this net amount of torque to the body will cancel the error out in time $\boldsymbol{\tau}$.

$$\Delta\varphi = \dot{\varphi}\tau + \frac{1}{2}(\ddot{\varphi}\tau^2) \rightarrow \ddot{\varphi} = \frac{2(\Delta\varphi - \dot{\varphi}\tau)}{\tau^2} \tag{10}$$
$$\widetilde{M} = I_{s,xx}\ddot{\varphi} = \frac{2I_{s,xx}(\Delta\varphi - \dot{\varphi}\tau)}{\tau^2} \tag{11}$$

This concept is used to initiate a method of finding an appropriate expression for tilting torque with respect to error and its rate. Then the procedure is continued using trial and error method to find the best suited amount of tilting torque in different conditions with keeping an eye on the maximum amount of torque allowed without violating the roll stability threshold which is about 4000 N.m of tilting torque.

We need an additional variable to find the total acting torque with this approach; we define a variable $\boldsymbol{M_{normal}}$ which interprets the amount of torque required to maintain the vehicle body in the current leaning angle. Since the controller is responsible for all of the interactions between the suspension and the body (sprung and unsprung masses), this amount of torque is needed in addition to $\widetilde{\boldsymbol{M}}$ to be implied to make the net inserted torque be equal to $\widetilde{\boldsymbol{M}}$. In other words, $\boldsymbol{M_{normal}}$ is only compensating the effect of LLT which will be eliminated in the desired tilt angle. Assuming $\boldsymbol{h_{s,CG}} \cong \boldsymbol{h_{CG}}$,

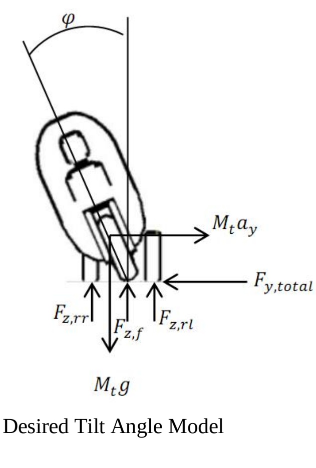


Fig4. Desired Tilt Angle Model

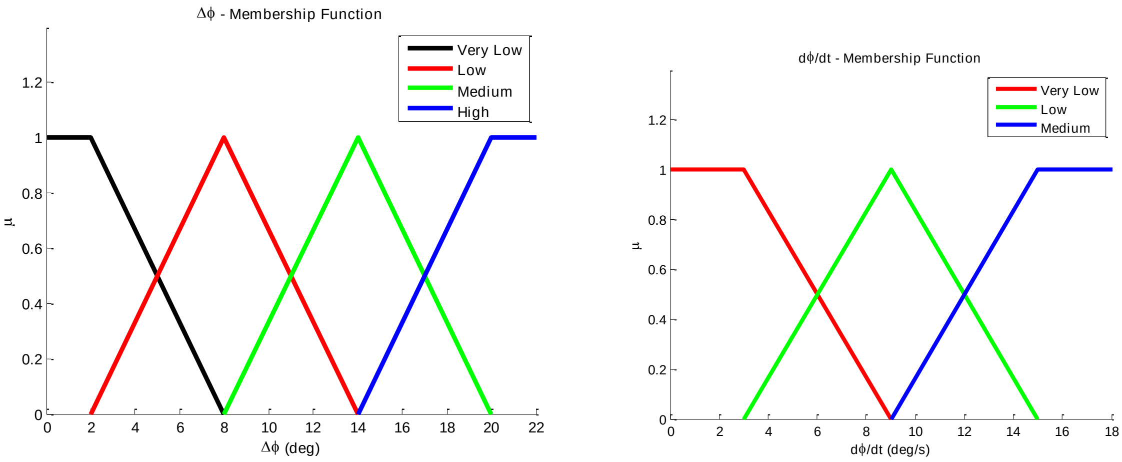


Fig5. Error and its Rate Membership Functions

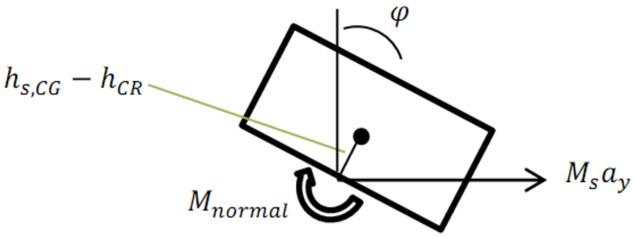


Fig6. Sprung Mass Model

$$M_{s,CG} = 0 \rightarrow M_{normal} + M_s g\left(h_{s,CG} - h_{CR}\right)\sin\varphi - M_s a_y\left(h_{s,CG} - h_{CR}\right)\cos\varphi = 0$$

$$M_{normal} = M_s\left(a_y\cos\varphi - g\sin\varphi\right)\left(h_{s,CG} - h_{CR}\right) \sim M_s\left(a_y\cos\varphi - g\sin\varphi\right) \quad (12)$$

And the total tilting moment is obtained as

$$M = M_{normal} + \widetilde{M} = M_{normal} + \frac{2I_{s,xx}(\Delta\varphi - \dot{\varphi}\tau)}{\tau^2} = M_{normal} + \frac{2I_{s,xx}(\mathrm{e} - \dot{\varphi}\tau)}{\tau^2} \quad (13)$$

Finally the fuzzy controller rule base is defined as follows:

$$If\ e\ is\ \mu^{VeryLow}\ and\ \dot{\varphi}\ is\ \mu^{VeryLow}\ then\ M = M_{normal} + f_1(e, \dot{\varphi}) = M_{normal} + \frac{2I_{s,xx}(\mathrm{e} - 0.5\dot{\varphi})}{0.5^2}$$

Method of multiplication is used for the ∧ operation [9]:

$$\mu_1 = \min\left(1, \mu_e^{Very\ Low} \times \mu_{\dot{\varphi}}^{Very\ Low}\right)$$

$$\mu_2 = \cdots$$

$$\mu_{12} = \min\left(1, \mu_e^{High} \times \mu_{\dot{\varphi}}^{Medium}\right)$$

Method of COA is used to de fuzzify the controller output [9]:

$$\widetilde{M} = \frac{\Sigma(\mu_i . f_i)}{\Sigma\mu_i} \quad (14)$$

4. Simulation Results

Response of the controlled TWV, uncontrolled TWV and conventional FWV are compared for step and slalom steering inputs in this section. Further simulations including lane change and impulse steering responses are discussed in [2].

All of the simulations are performed with constant longitudinal vehicle speed of 72 km/h unless otherwise is mentioned. Lateral force, lateral acceleration, slip angles, LLT, roll and tilt angles, tilt angle error, tilting torque and the vehicle path are considered as assessment criteria and are discussed in each case. Purpose of the following analyses is to compare the above variables in the same vehicle path for each steering type. In discussions, the term “Ability to Generate Lateral Force” is intended to mean the “ratio of generated lateral force to the steering angle” equivalent to the “lateral acceleration gain”.

Table 1- Fuzzy Rule Base

Rule #	Condition	$\boldsymbol{\tau}$ in moment equation ($\tilde{\boldsymbol{M}} = \boldsymbol{f_i}(\boldsymbol{e}, \dot{\boldsymbol{\varphi}})$)
1*	$(\boldsymbol{e\ is\ Very\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Very\ Low})$	$\boldsymbol{\tau = 0.5}$
2	$(\boldsymbol{e\ is\ Very\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Low})$	$\boldsymbol{\tau = 9.3}$
3	$(\boldsymbol{e\ is\ Very\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Medium})$	$\boldsymbol{\tau = 16.0}$
4	$(\boldsymbol{e\ is\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Very\ Low})$	$\boldsymbol{\tau = 1.0}$
5	$(\boldsymbol{e\ is\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Very\ Low})$	$\boldsymbol{\tau = 0.9}$
6	$(\boldsymbol{e\ is\ Low}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Medium})$	$\boldsymbol{\tau = 18}$
7	$(\boldsymbol{e\ is\ Medium}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Very\ Low})$	$\boldsymbol{\tau = 1.5}$
8	$(\boldsymbol{e\ is\ Medium}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Low})$	$\boldsymbol{\tau = 1.0}$
9	$(\boldsymbol{e\ is\ Medium}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Medium})$	$\boldsymbol{\tau = 23.0}$
10	$(\boldsymbol{e\ is\ High}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Very\ Low})$	$\boldsymbol{\tau = 2.5}$
11	$(\boldsymbol{e\ is\ High}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Low})$	$\boldsymbol{\tau = 1.6}$
12	$(\boldsymbol{e\ is\ High}) \wedge (\dot{\boldsymbol{\varphi}}\ \boldsymbol{is\ Medium})$	$\boldsymbol{\tau = 1.2}$

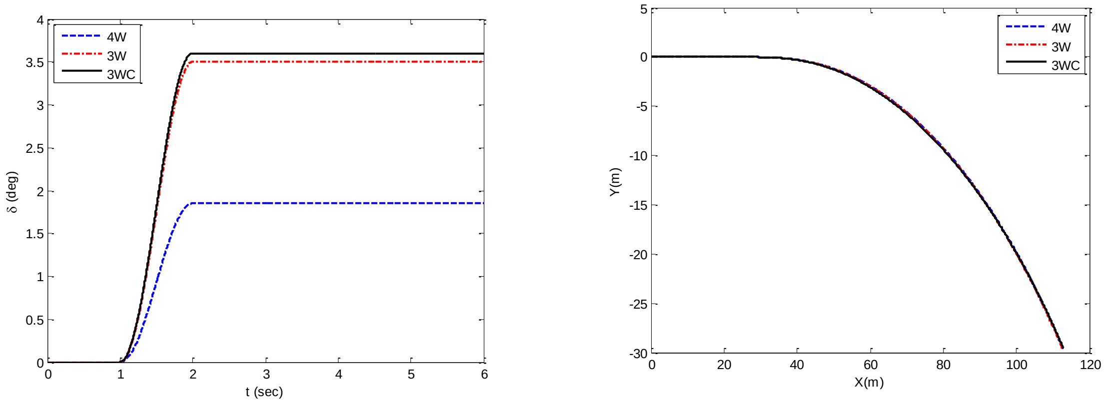


Fig7. Steering Angle and Vehicle Path

4.1. Step Steering

By this analysis, we want to simulate the situation in which the driver intends to pass a simple turn with a constant speed and steering angle as shown in the following figure:

Uncontrolled TWV show a lot more under steering than the FWV which is also predictable from the basic physics of the TWVs with a single front wheel. The controlled TWV shows even more under steering with respect to the uncontrolled one.

The FWV needs less slip angle to produce the required lateral force and its ability to generate lateral force is higher than the TWVs as expected. This means that the FWV is still able to generate more lateral force in higher slip angles despite the TWVs.

The controlled TWV is dealing with lower slip angles in all tires than the uncontrolled one which implies the role of camber angles in lateral force production. Hence, it can be concluded that the controlled TWV possesses more ability to generate lateral force than the uncontrolled one due to the contributing camber angles.

According to the same vehicle path and speed, we may expect similar lateral acceleration for three vehicles, but the behavior is not exactly the same and the controlled TWV seems to produce more lateral acceleration in a shorter time than two other vehicles. Faster changes in normal tire forces in controlled

TWV – and consequently faster settlement in those conditions - can be the cause of this phenomenon.

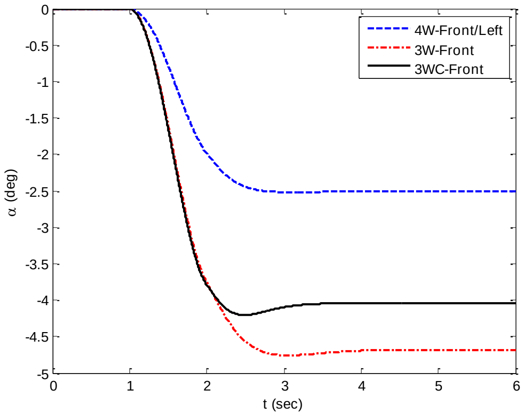


Fig8. Front Tire Slip Angles

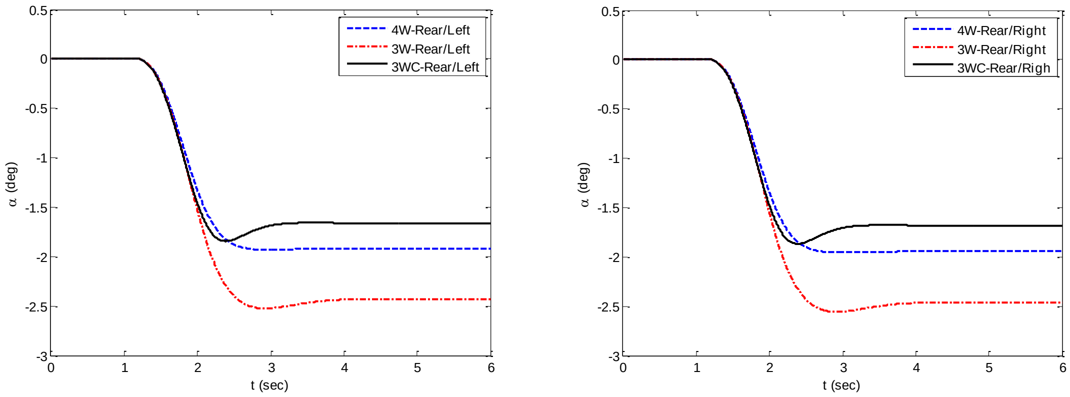
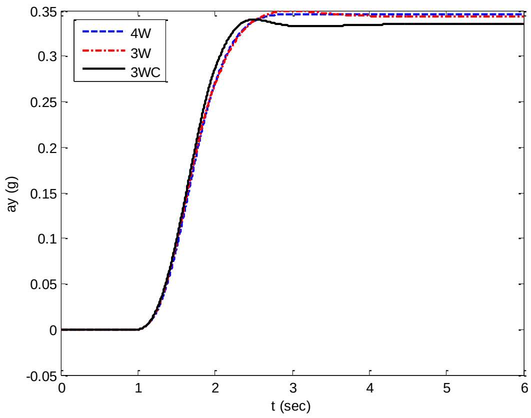


Fig9. Rear Tire Slip Angles



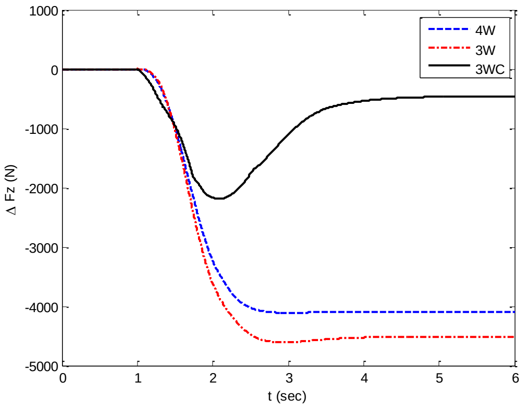


Fig10. Lateral Acceleration (left)

Fig11. Lateral Load Transfer (right)

Our important purpose in using tilt control was to minimize the LLT. In the above figure it is shown that the controlled TWV experienced a very low amount of LLT in steady state compared to the two normal vehicles. The important note is that the tilting torque causes a bit higher amount of LLT at first, but gradually this LLT is reduced dramatically with respect to normal behavior which shows a good success in control strategy although the steady state error keeps the LLT from becoming identically zero.

In this analysis, the uncontrolled TWV is near the roll-over threshold according to the vehicle parameters – nominal normal load on each rear wheel is about 5400 N - while the controlled TWV is far away from instability due to LLT reduction.

High amount of roll means uncomfortable passenger conditions and high torque exerted to the sprung mass while causing difficulties for the driver to control the vehicle properly. The roll angle in uncontrolled TWV is about twice the amount of that in FWV; on the other hand, the controlled TWV leans towards inside of the turn and makes passengers not to feel any lateral force on their bodies, but they feel pressed to their seats instead of that. This condition can also give a kind of sporty feeling to the passengers which might not be a desired case for elderly people.

Tilting torque in the controlled TWV is smooth and no fluctuation exists which is considered a good control feature. The tilting torque has not become finally zero because of the steady-state error in tilting angle.

As mentioned previously, the uncontrolled TWV is extremely unstable and rolls over easily. The stability threshold for this vehicle for step response analysis is about 72 km/h speed with a 3.5 degree steering angle. Thus, in the following section we compare the step response for the controlled TWV and the FWV at a higher speed (110 km/h) to assess the ability of the controlled TWV.

The following vehicle path is generated for the two vehicles:

As we mentioned before, the controlled TWV is much more under steer than the FWV which is also obvious from the above figure. This feature is an inherent property of TWVs with one front wheel and it is almost not affected by the use of tilt control system.

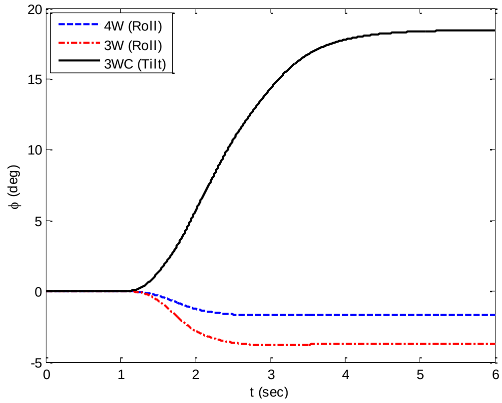


Fig12. - Roll / Tilt Angles

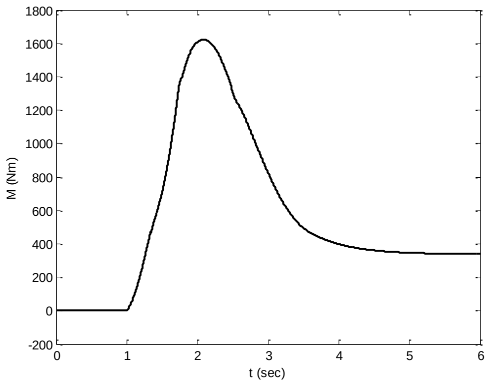


Fig13. Tilting Torque

Control system has done well and the LLT is reduced dramatically compared to that of the FWV, but the existence of steady-state error keeps the LLT to become identically zero in this case as well.

Tilting torque figure shows a rapid response and smooth behavior of the control system as well as the desirable overshoot with respect to the maximum allowable tilting torque guaranteeing the roll-over stability (about 4000 Nm).

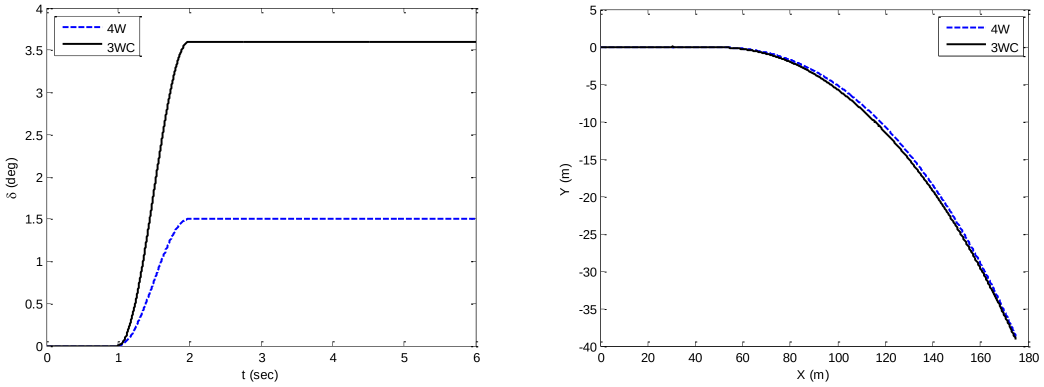


Fig14. Steering Angle and Vehicle Path

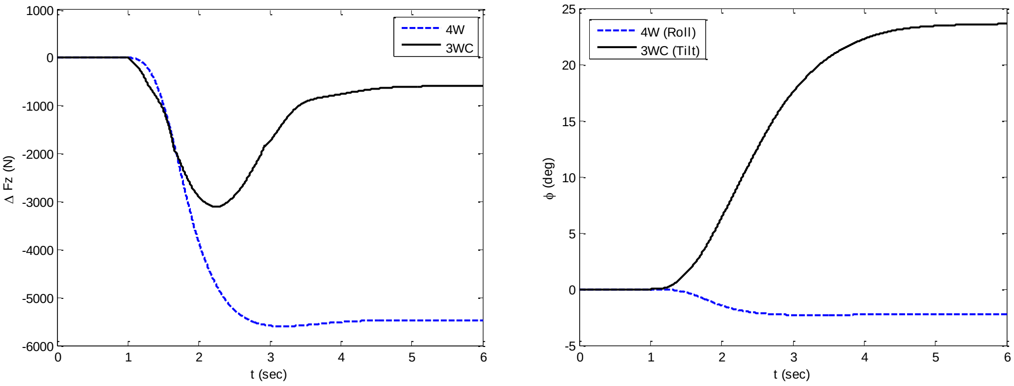


Fig15. Lateral Load Transfer and Roll/Tilt Angle

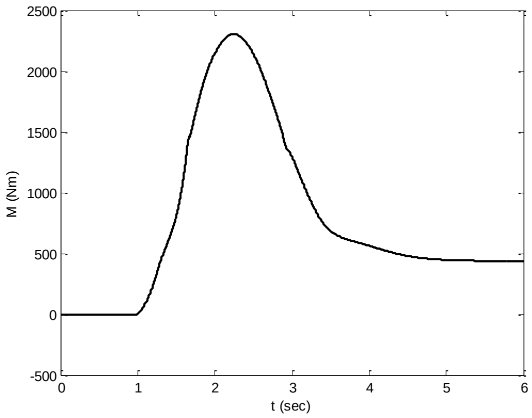


Fig16. Tilting Torque

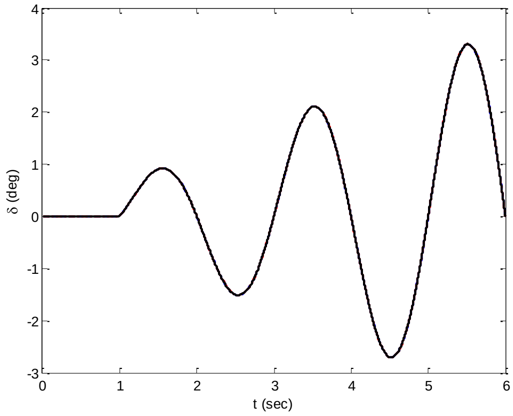


Fig17. Steering Angle

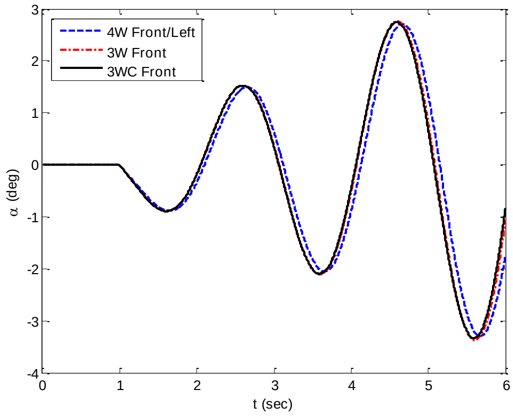


Fig18. Front Tire Slip Angles

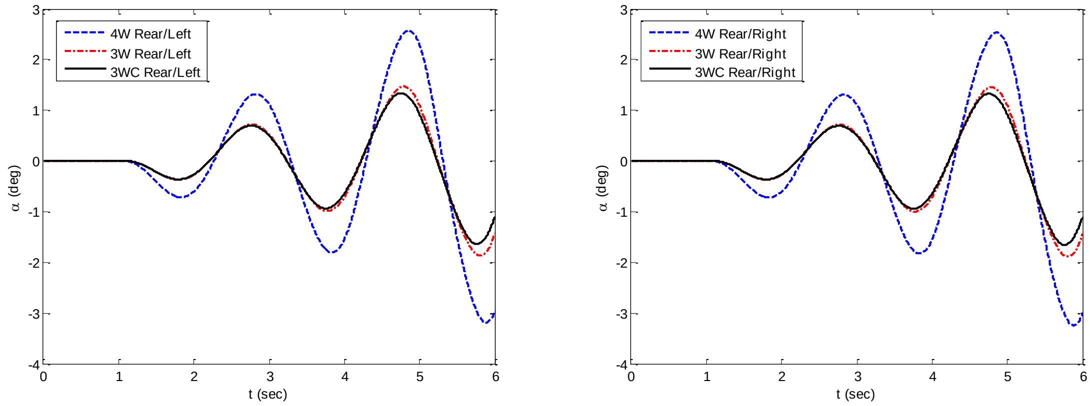


Fig19. Rear Tires Slip Angles

4.2. Slalom Steering

In this condition, the vehicle path is not of great importance and we can use a similar steering input for all three vehicle types in order to compare them in this critical state.

In this maneuver the tilt control system seems to have almost no effect on lateral behavior of the TWV.

LLT is dramatically reduced using the tilt control system while not affecting the lateral behavior of

TWV which is quite desirable as far as our control purpose is concerned. This analysis depicts the ability of the control system to maintain its performance and stability in the most critical steering condition Tilting torque is applied smoothly with a good response time in this critical condition. The important point we can get from the above figure is that if we continue applying the critical steering input we will face more and more tilting torque which strongly implies the need of the control system to some kind of torque limiter in order to keep the vehicle stable.

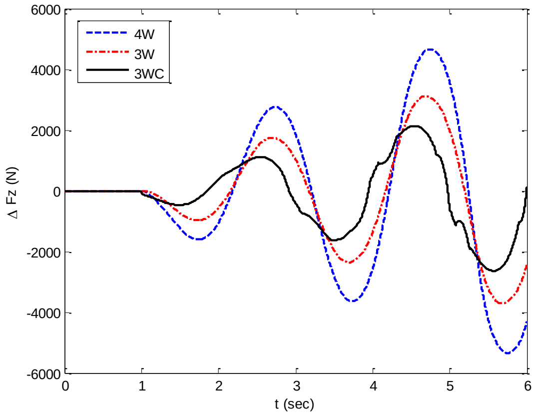


Fig20. Lateral Load Transfer

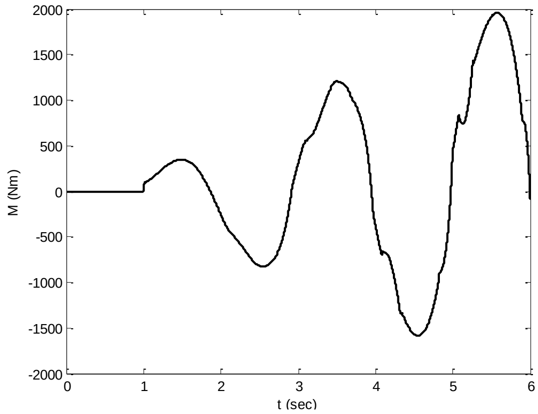


Fig21. Tilting Torque

5. Conclusion

According to the simulation results, the tilt control system is able to eliminate the roll-over stability problems of the normal TWV properly by reducing the LLT and provides much safer performance for such vehicles in normal speeds and urban usage.

The steady-state error of the designed fuzzy control system is very low and less than 1 degree which is considered acceptable compared to the amount of tilt angles associated with proposed maneuvers.

It is worth to mention that this kind of vehicle dynamic control requires a torque and tilt angle limits according to slalom analysis result and in practice it is necessary to perform enough safety tests before actual use of the system.

Tilt control system didn’t show notable effects on lateral behavior of the TWV, but has upgraded its roll-over stability greatly which means that the vehicle is much more stable in similar lateral conditions in controlled mode. This implies that if the vehicle is equipped with tires with more stiffness, it can achieve more lateral accelerations and better handling without having any concern about its roll-over stability which is the most critical weak spot of the TWVs.

The controlled TWV provides the passengers with much more lateral convenience; on the other hand, it also provides a sense of sportiness which may limit the target market.

Response time of the fuzzy control seems to be acceptable in all analyses made in this paper even in critical ones, but it is still worth to mention that by use of some kind of STC it seems possible to improve the response time by taking a steering wheel rotation feedback for instance.

In the controlled TWV, a portion of lateral force is generated by the favorable camber angle of the wheels produced due to vehicle body tilting. This feature depicts the combination of lateral abilities of the car and motorcycle in the controlled TWV as mentioned before. In motorcycles, most of the lateral force is generated due to the camber angles.

The ability to generate lateral force is much more in FWV in all conditions. TWVs show a kind of delay in generating lateral force making them less handled compared to FWV. Normal TWV is a lot weaker than a FWV because of its roll-over tendency which is dramatically reduced using tilt control system enabling the designers to use stiffer tires and minimizing the difference between lateral abilities of the FWVs and TWVs.

Suggestions for Future Studies:

In this paper only a fuzzy control is tried for the proposed system; other types of non-linear control can be used as well and results can be compared.

Effect of DTC and STC combination can be investigated to see if the response time is optimized or not.

Detailed mechanism of tilt system can also be investigated in future works which also eases the use of such tilt controllers.

In this paper, only the TWV with one front wheel is simulated and analyzed, so the other type of TWVs with single rear wheel can also be equipped with a tilt control to compare the behaviors.

Method of combining the tilt control system with other safety systems such as ESP can also be investigated.

Fuzzy rule base can also be manipulated by concentration on improving the system behavior around zero tilt angle error point.

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APPENDIX:

Nomenclature:

Parameter / Variable	Description	Parameter / Variable	Description
M_t	Total Mass of the Vehicle	$F_{y,f}$	Single Front Tire Lateral Force
M_s	Sprung Mass	$F_{y,rl}$	Left Rear Tire Lateral Force
T_r	Rear Track Width	$F_{y,rr}$	Right Rear Tire Lateral Force
L_f	Distance between Front Axle and CG	$F_{z,f}$	Single Front Tire Normal Force
L_r	Distance between Rear Axle and CG	$F_{z,rl}$	Left Rear Tire Normal Force
L	Wheelbase	$F_{z,rr}$	Right Rear Tire Normal Force
k_s	Spring Stiffness	$M_{z,f}$	Single Front Tire Self-Aligning Torque
b_s	Damping Coefficient of Shock Absorber	$M_{z,rl}$	Left Rear Tire Self-Aligning Torque
$I_{s,xx}$	Sprung Mass Moment of Inertia about Longitudinal Axis	$M_{z,rr}$	Right Rear Tire Self-Aligning Torque
I_{zz}	Vehicle Moment of Inertial about Normal Axis	δ	Steering Input
h_{CG}	CG Height	M	Tilt Controlling Torque
h_{CR}	Center of Roll Height	$e = \Delta\varphi$	Tilt Angle Error
$h_{s,CG}$	Sprung Mass CG Height	M_{normal}	Required Torque to Maintain Current Tilt Angle
φ	Roll Angle	$\widetilde{M}$	Additional Controlling Torque
ψ	Yaw Angle	γ_f	Single Front Wheel Camber Angle
α_f	Single Front Wheel Slip Angle	γ_{rl}	Left Rear Wheel Camber Angle
α_{fl}	Left Front Wheel Slip Angle	γ_{rr}	Right Rear Wheel Camber Angle
α_{fr}	Right Front Tire Slip Angle	X	Vehicle Global Coordinate
α_{rl}	Left Rear Tire Slip Angle	Y	Vehicle Global Coordinate
α_{rr}	Right Rear Tire Slip Angle	g	Gravitational Acceleration

Vehicle Parameters:

Parameter	M_t	M_s	$I_{s,xx}$	I_{zz}	T_r	L_f	L_r	h_{CG}	h_{CR}	k_s	b_s
Value	1349 kg	1176 kg	496 kg.m2	2249 kg.m2	1.483 m	1.053 m	1.559 m	0.605 m	0 m	47000 N/m	3000 N.s/m