Robust optimal multi-objective controller design for vehicle rollover prevention

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Abstract

Robust control design of vehicles addresses the effect of uncertainties on the vehicle's performance. In present study, the robust optimal multi-objective controller design on a non-linear full vehicle dynamic model with 8-degrees of freedom having parameter with probabilistic uncertainty considering two simultaneous conflicting objective functions has been made to prevent the rollover. The objective functions that have been simultaneously considered in this work are, namely, mean of control effort (MCE) and variance of control effort (VCE).The nonlinear control scheme based on sliding mode has been investigated so that applied braking torques on the four wheels are adopted as actuators. It is tried to achieve optimum and robust design against uncertainties existing in reality with including probabilistic analysis through a Monte Carlo simulation (MCS) approach in multi-objective optimization using the genetic algorithms. Finally, the comparison between the results of deterministic and probabilistic design has been presented. The comparison of the obtained robust results with those of deterministic approach shows the superiority robustness of probabilistic method.

***Keywords**: Robust design, Vehicle rollover, Uncertainty, Dynamic model, Pareto optimization, Control, Monte Carlo*

1. . Introduction

The rollover of vehicle has attracted increasing attention in the field of ground transportation safety problem. During the last decades, The growing popularity of utilizing Sport Utility Vehicles (SUVs) with higher center of gravity than that of passenger vehicles is responsible for a more precise investigation in manufacturing regulations aimed at reducing rollover fatalities because of high cost of production and manufacturing, together with their higher rollover tendency. According to the published reports by National Highway Traffic Safety Administration (NHTSA), the most important portion of fatal injuries of road accidents was caused by vehicle rollover [1]. In 2002, it was reported that approximately 11 million of land transportation passengers in roads and highways have been exposed to an accident, in which 3% of these have been caused by vehicles rollover. Moreover, 33% of the number of passengers lost their lives due to the occurrence of rollover, which is considerable in comparison with other types of accidents [1].

In this way, a large number of different strategies for vehicle rollover mitigation have been proposed in the literature. Abe et al. used experimental validation of side-slip estimation to show the effects of side-slip control by direct yaw moment on improving vehicle motion stabilities. They showed that side-slip control by direct yaw control (DYC) can stabilize vehicle motion much better than 4-wheel steering (4WS) because vehicle loses its stability due to deterioration of rear tire characteristics [2]. Bouton et al proposed a rollover risk indicator dedicated to off-road vehicles, taking into account the environment properties and more particularly the grip condition and its variation. Their controller is based on the prediction of the lateral load transfer relying on 3-DOF vehicles model including sliding effects and performances of this indicator are demonstrated using the multi-body dynamic simulation software Adams [3]. Different investigations were carried out in order to design rollover controller by considering parameter variations and tried to present robust controller which can overcome uncertainty in vehicle model [4-6]. Yim used a 2-DOF vehicle model for designing a controller that uses active anti-roll bar and electronic stability program (ESP) with longitudinal speed control for preventing rollover based on linear quadratic static output feedback [7].

Accordingly, prevention of vehicle rollover is so crucial in designing more effective active safe control systems. There are many theoretical evidence based on linear and single-track models with low degree of freedom that are widely utilized in the literature [4, 7-11]. However, these models are based on some assumptions and approximations which have unlikely no credit during the extreme maneuvers where the non-linear conditions, tire properties and dynamical conditions of the vehicle should be considered. Also, load transfer and roll dynamics behavior which is done during these conditions can't be modeled by the single track model. With respect to the mentioned reasons and in order to resolve the above limitations, the modeling has been made as non-linear and two-track.

In this paper, multi-objective Pareto genetic algorithms, with a diversity ε-elimination algorithm is used in conjunction with MCS in order to Pareto optimization of controller for rollover prevention an uncertain 8-degree of freedom vehicle dynamic model subjected to probabilistic variations of the total mass of vehicle. Considering the non-linearity of characteristics and parameters related to the vehicle dynamics as well as the uncertainties in the reality, a non-linear controller with increased robustness should be designed for this aim. The obtained results demonstrate that compromise can be readily accomplished using graphical representations of trade-offs between the conflicting objectives of the statistical measures of mean and variance of the control effort. At the end, obtained results demonstrate that the strategy is significantly capable of preventing vehicle rollover during extreme fishhook maneuver and also are compared with those obtained from the determinate design approach.

2. Stochastic Robust Analysis

In real engineering practice, there exist varieties of typical sources of uncertainty that have to be compensated through a robust design approach. Two categorical types of uncertainty, namely, structured uncertainty and unstructured uncertainty are generally used in classification. The structured uncertainty concerns about the model uncertainty due to unknown values of parameters in a known structure. In conventional optimum system design, uncertainties are not addressed and the optimization process is accomplished deterministically. In fact, it has been shown that optimization without considering uncertainty generally leads to non-optimal and potentially high risk solution [12]. Generally, there exist two approaches addressing the stochastic robustness issue, namely, robust design optimization (RDO) and reliability-based design optimization (RBDO) [13]. Monte Carlo simulation (MCS) has also been used to verify the results of other methods in RDO or RBDO problems when sufficient number of sampling is adopted [14-15]. Monte Carlo simulation (MCS) is a direct and simple numerical method but can be computationally expensive. Let X be a random variable, then the prevailing model for uncertainties in stochastic randomness is the probability density function (PDF), $f_X(x)$ or equivalently by the cumulative distribution function (CDF), $F_X(x)$, where the subscript X refers to the random variable. This can be given by:

$$F_X(x) = \Pr(X \le x) = \int_{-\infty}^{x} f_X(x)dx \tag{1}$$

In which $\Pr(.)$ denotes the probability that an event ($X \le x$) will occur. Some statistical moments such as the first and the second moment, generally known as mean value (also referred to as expected value) denoted by $E(X)$ and variance denoted by $\sigma^2(X)$, respectively, are of the most important ones. They can also be computed by:

$$E(X) = \int_{-\infty}^{\infty} x dF_X x = \int_{-\infty}^{\infty} x f_X(x)dx \tag{2}$$

and

$$\sigma^2(X) = Var(X) = \int_{-\infty}^{\infty} (x - E(X))^2 dF_X(x) = \int_{-\infty}^{\infty} (x - E(X))^2 f_X(x)dx \tag{3}$$

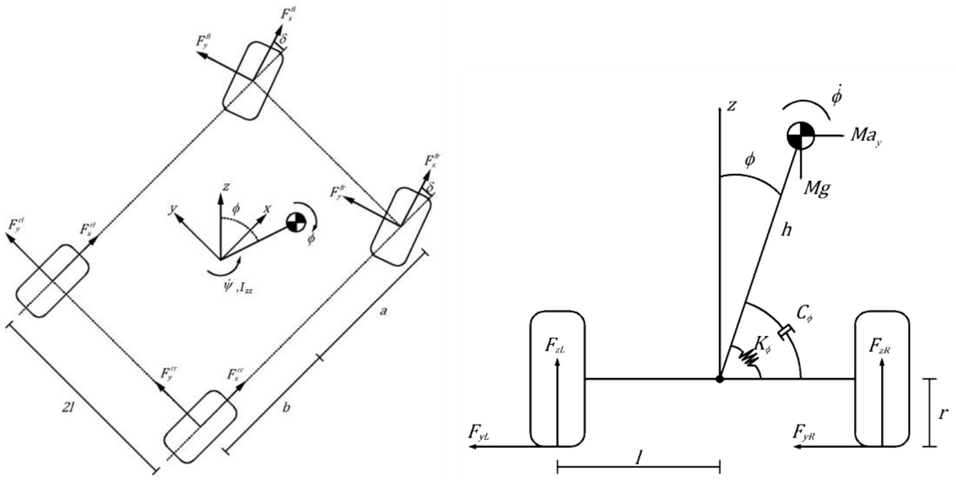
In the case of discrete sampling, these equations can be readily represented as:

$$E(X) \cong \frac{1}{N}\sum_{i=1}^{N} x_i \tag{4}$$

and

$$\sigma^2(X) = Var(X) \cong \frac{1}{N-1}\sum_{i=1}^{N} (x_i - E(X))^2 \tag{5}$$

Where x_i is the ith sample and N is the total number of samples. In the case of robust control design of this work, the mean and the variance of the control effort are computed by MCS approach using equations (4) and (5), respectively. The statistical measures of both mean and variance depend on the number of samples in the region of the space of uncertain parameters. Evidently, such estimations approach to the actual value in the limit as $N \to \infty$ [16-17]. However, there have been many research activities on sampling techniques to reduce the number of samples keeping a high level of accuracy. Alternatively, the quasi-MCS has now been



(a) (b)

Fig1. Vehicle dynamic model. (a) Top view (b) Behind view

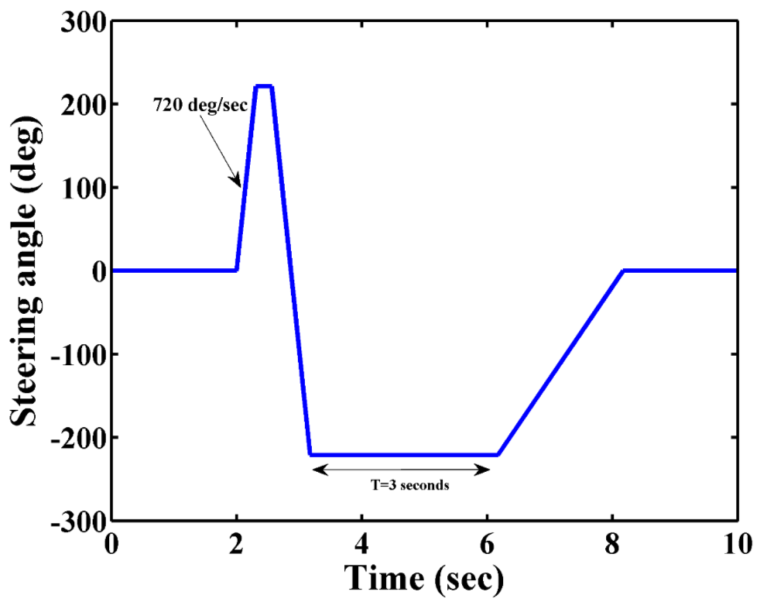


Fig2. Fixed fishhook maneuver

Increasingly accepted as a better sampling technique which is also known as Hamersley Sequence Sampling (HSS) [18]. In a multi-objective optimization of a RDO problem presented in this paper, the conflicting robust metrics of mean and variance of control effort should be minimized simultaneously.

3. System dynamics

3.1. Vehicle dynamic model

In order to describe a schematic modeling of a nonlinear two-track full vehicle dynamic model including roll dynamics and the model in vertical plane, Fig. 1 is displayed. The suspension system is modeled as a torsional spring and damper acting around the roll axis, in which C_φ and K_φ express roll damping coefficient and roll stiffness coefficient, respectively. The resulting model has eight degrees of freedom, namely rotational speed of the wheels, translational motion along the x and y axis, as well as rotational motion around the x axis (roll) and the z axis (yaw). The governing equations of angular and translational motion of the vehicle model are as follows [19]:

$$\ddot{\phi} = \frac{F_{yT} h\, cos\phi + mgh\, sin\phi - C_\phi \phi - K_\phi \dot{\phi} + \dot{\psi}^2 (I_{yy} - I_{zz}) sin\phi\, cos\phi}{I_{xx}} \tag{6}$$

$$\ddot{\psi} = \frac{M_T - F_{xT} h\, sin\phi - 2(I_{yy} - I_{zz}) sin\phi cos\phi \dot{\phi}\dot{\psi}}{I_{yy} sin^2\phi + I_{zz} cos^2\phi} \tag{7}$$

$$\dot{u} = \frac{F_{xT}}{m} + \upsilon\dot{\psi} - hsin\phi\left(\frac{M_T - F_{xT} h\, sin\phi - 2(I_{yy} - I_{zz}) sin\phi cos\phi \dot{\phi}\dot{\psi}}{I_{yy} sin^2\phi + I_{zz} cos^2\phi}\right) \tag{8}$$

$$\dot{\upsilon} = \frac{F_{yT}}{m} - u\dot{\psi} - hsin\phi cos\phi \dot{\psi}^2 + \frac{h}{I_{xx}}(F_{yT} h\, cos\phi + mgh\, sin\phi - C_\phi \phi - K_\phi \dot{\phi} + \dot{\psi}^2 (I_{yy} - I_{zz}) sin\phi\, cos\phi) \tag{9}$$

In which m, h, F_{xT}, F_{yT}, M_T and I_{ii} denote the vehicle mass, the height of the center of gravity above the roll axis, total longitudinal force, total lateral force, total moment and the moment of inertia about the each axis, respectively. And also the governing equations for motions of the wheel [20-21] are given by:

$$rF_x^{fl} - T_b^{fl} = I_w \dot{\omega}^{fl} \tag{10}$$

$$rF_x^{fr} - T_b^{fr} = I_w \dot{\omega}^{fr} \tag{11}$$

$$rF_x^{rl} - T_b^{rl} = I_w \dot{\omega}^{rl} \tag{12}$$

$$rF_x^{rr} - T_b^{rr} = I_w \dot{\omega}^{rr} \tag{13}$$

Where r, I_w, $T_b[T_b^{fl}\, T_b^{fr}\, T_b^{rl}\, T_b^{rr}]$, ω and F_x outline the wheel radius, the total moment of inertia of the wheel, the braking torque which must be determined from the control law (control inputs), the angular velocity of wheel and the longitudinal tire force, correspondingly. Henceforth, the superscripts and the subscripts rr, rl, fr and fl indicate the rear right, rear left, front right and front left, respectively.

3.2. Nonlinear tire force model

In order to simulate the nonlinear behavior of the tire Magic Formula [19] and [22] is employed to calculate the tire longitudinal and lateral forces considering combine slip, i.e. simultaneous occurrence of lateral and longitudinal slips. Longitudinal force F_x is generated by the longitudinal slip κ, and the lateral force F_y is generated by the lateral slip α. The general form of the Magic Formula [19] and [22] that holds for given values of vertical load, slip components and camber angle is given by:

$$y = D \sin[C \arctan\{B\, z - E(arctanBz)\}] \tag{14}$$

Where z is the input variable longitudinal slip κ or the lateral slip α, y is the output variable Fx, Fy or possibly Mz, B is the stiffness factor, C is the shape factor, D is the peak value and E is the curvature factor.

4. Control system design

In this section, a nonlinear multiple-input multiple-output (MIMO) control system based on sliding mode [23-24] is presented to prevent vehicle rollover. Vehicle rollover must be prevented by minimum mean and variance of braking torque as the control effort. To achieve these aims, according to the conflict of these objective functions, multi-objective uniform-diversity genetic algorithm method called MUGA [17] and [25] is utilized.

From Eqs. (6)- (13), the state-space representation of the vehicle dynamic model can be expressed as follows:

$$\dot{x} = f(x) + bu \tag{15}$$

Where

$x = [x_1\, x_2\, x_3\, x_4\, x_5\, x_6\, x_7\, x_8\, x_9\, x_{10}]^T \in \Re^{10}$ is the vehicle states vector which are: $x_1 = \phi$: roll angle,

$x_2 = \dot{\phi}$: roll angular velocity, $x_3 = \psi$: yaw angle, $x_4 = \dot{\psi}$: yaw angular velocity, $x_5 = u$: longitudinal velocity, $x_6 = v$: lateral velocity, $x_7 = \omega^{fl}$: angular velocity of the fl wheel, $x_8 = \omega^{fr}$: angular velocity of the fr wheel, $x_9 = \omega^{rl}$: angular velocity of the rl wheel, $x_{10} = \omega^{rr}$: angular velocity of the rr wheel. $u = [T_b^{fl}\ T_b^{fr}\ T_b^{rl}\ T_b^{rr}\ ]^T \in \Re^4$ is the vector of the system control inputs. b is matrices with appropriate dimension. According to the SMC theory, the switching surface S is defined as:

$$S = \tilde{x} + k_T \int \tilde{x} \tag{16}$$

Where $k_T = diag([k_{T_1}\ k_{T_2}\ k_{T_3}\ \ k_{T_4}\ k_{T_5}\ k_{T_6}\ k_{T_7}]) \in \Re^{7\times 7}$ is SMC gain. The error between the actual value and the desired value can be written as:

$$\tilde{x} = x_c - x_c^d \tag{17}$$

Where $x_c = [x_1\ x_2\ \ x_4\ \ x_7\ x_8\ x_9\ x_{10}]^T \in \Re^7$ the control states vector and also the desired states vector is defined as $x_c^d \in \Re^7$. The switching control law tries to reach the system's states on the sliding surface and a control law could be formulated that would keep $\dot{S} = 0$. The time derivative of S is given by:

$$\dot{S} = \dot{\tilde{x}} + k_T \tilde{x} \tag{18}$$

According to Eq. (15), by substituting $\dot{\tilde{x}} = f(\tilde{x}) + bu$ into Eq. (18) yields:

$$\dot{S} = f(\tilde{x}) + bu + k_T \tilde{x} \tag{19}$$

Hence, the control law is obtained as follows:

$$u = -b^{-1} k_T \tilde{x} - b^{-1} f(\tilde{x}) \tag{20}$$

5. Pareto robust optimization of controller for the vehicle dynamic model with probabilistically uncertain parameter

In this section, multi-objective Genetic Algorithm (GA) with a diversity ε-elimination algorithm is used for robust pareto optimal sliding mode controller design for rollover prevention of 8-degrees of freedom vehicle dynamic model with probabilistic uncertainty. This robust optimal design consists of finding the vector of design variables $[k_{T1}, k_{T2}, k_{T3}, k_{T4}, k_{T5}, k_{T6}, k_{T7}]$ so that two conflicting objective functions, namely, mean of the control effort (MCE) and variance of the control effort (VCE) are minimized simultaneously.

It should be noted that the control effort (summation of four applied braking torques on the four wheels) as follows:

$$CE = \sum_{i=1}^{4} u_i \tag{21}$$

where $u_1 = \int T_b^{fl}\, dt$, $u_2 = \int T_b^{fr}\, dt$, $u_3 = \int T_b^{rl}\, dt$ and $u_4 = \int T_b^{rr}\, dt$.

It should be considered that the statistical moments of the control effort (CE) of equation (21) are computed using equations (4) and (5) via the MCS approach. Evidently, this is an optimization robust problem with two cost functions and seven decision variables. The range of input variables is set to $k_{Ti} \in [0,1000] (i = 1-7)$. Also, the input values of fixed parameters of the vehicle model are presented at Table 1. In the simulation of this study, the steering input is fixed fishhook maneuver with the maximum angle of 221º, as displayed in Figure 2 [26]. The initial speed of vehicle is set to 80 km/h.

The uncertain parameter of the vehicle dynamic model that has been selected is m (the vehicle mass). The uncertainties of mentioned parameter are assumed according to the Gaussian probabilistic distribution function within the limits of ±10 percent of its corresponding nominal value.

The multi-objective uniform-diversity genetic algorithm (MUGA) is now used for Pareto robust optimization of SMC for the vehicle rollover prevention. In order to robustly design of sliding mode controller for rollover prevention of a full vehicle dynamic model from the multi-objective point of view, a population of 80 individuals with a crossover probability of 0.95 and mutation probability of 0.1 has been used in 240 generation for which no future improvement has been achieved. The robust optimization process is accomplished by 15 Monte Carlo evaluations for each candidate individual during the evolutionary process.

Table 1. . The numerical values of pertinent parameters of the vehicle model

R	0.36m	m	1609 kg
H	0.51m	I_w	0.9 kg m^2
A	1.05m	I_{xx}	377 kg m^2

b	1.569m	I_{yy}	1765 kg m^2
K_ϕ	60000 N.m/rad	I_{zz}	1765 kg m^2
C_ϕ	1000 N.m.s/rad	l	0.882 m

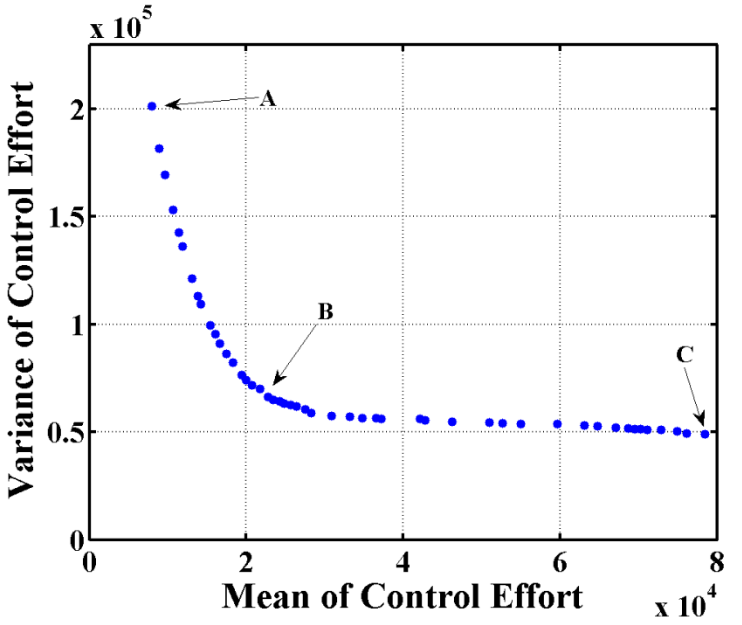


Fig3. Pareto front of objectives MCE and VCE by hybrid use of MUGA and MCS

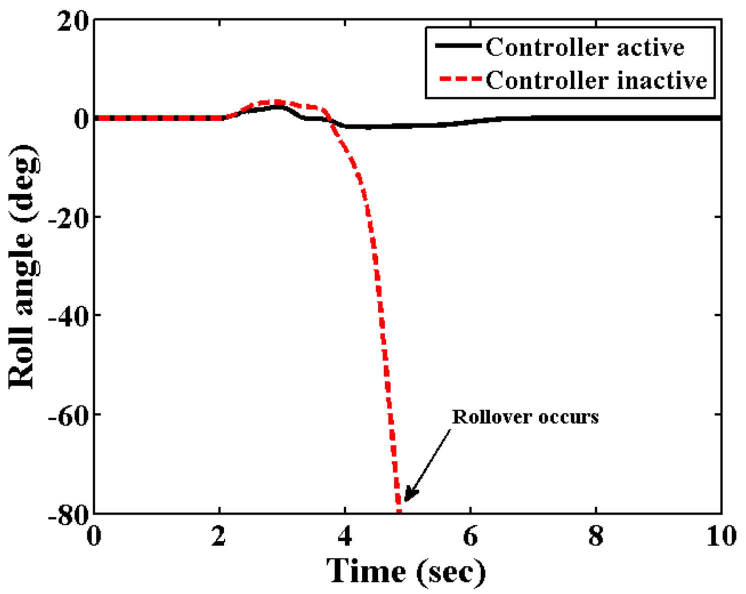


Fig4. Roll angle of the of vehicle model corresponding to the trade-off point B of the Pareto front shown in Fig. 3.

Table 2. The objective functions and their associated design variables for the optimum points B

Mean of control effort	Variance of control effort	k_{T_1}	k_{T_2}	k_{T_3}	k_{T_4}	k_{T_5}	k_{T_6}	k_{T_7}
22237	63052	302.08	146.62	106.23	359.47	332.64	356.07	264.27

Table 3. Statistical metrics of the objective functions associated with design variables point G (deterministic design) and trade-off design point B (robust design) computed by MCS (10000 samples)

Design Points	Statistical metrics	Summation of values for 4 control inputs
Design point B (robust design)	Mean of control effort	22237
	Variance of control effort	63052
Design point G (deterministic design)	Mean of control effort	138871
	Variance of control effort	6421063

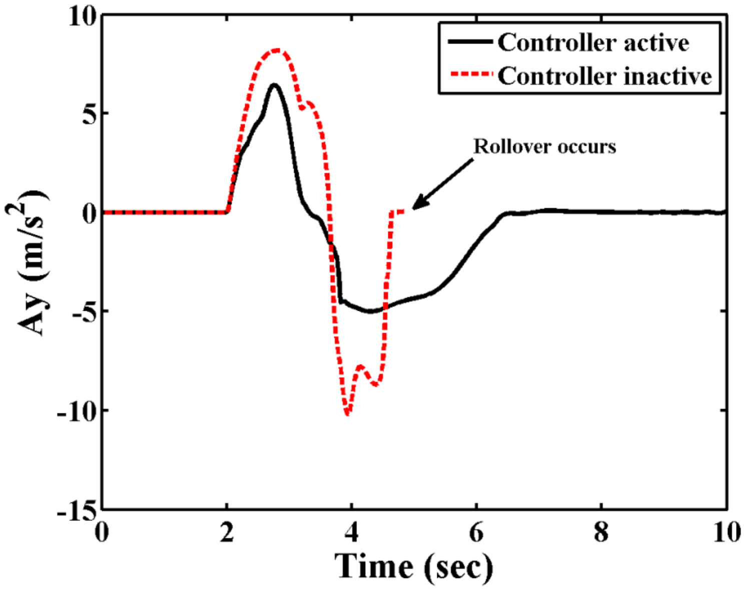


Fig5. Lateral acceleration of the of vehicle model corresponding to the trade-off point B of the Pareto front shown in Fig. 3.

Accordingly, a trade-off design point (point B) is suggested with respect to the conflicting objectives have been obtained, as demonstrated in Fig. 3 in the plane of objective functions as the Pareto front. The values of objective functions and their associated design variables of point B are shown in Table 2. Figure 4 and Figure 5 depict the vehicle roll angle and the vehicle lateral acceleration with and without controller, respectively. The time response of vehicle roll angle demonstrate that rollover occurs after approximately 4.7 seconds as the controller is deactivated, just after the maximum value of the second steering action is attained. With careful observation, the time response of vehicle lateral acceleration also demonstrates that the maximum lateral acceleration reduces around 22 percentages compared to deactivated controller case. Fig. 6 shows the controller inputs of the trade-off design point B under the fishhook maneuver. It should be noted that the highest value of obtained braking torque is lower than that of Ref [4] and [21]. The trajectories of the vehicle corresponding to the trade-off robust design point B during the fishhook maneuver with and without control are shown in Figure 7. It can be seen that the vehicle follows the desired fishhook trajectory when the controller is active. Moreover, the

Pareto fronts and controller inputs of the best individual in term of the control effort designed deterministically (design point G) are, correspondingly, shown in Figs. 8 and 9. It is very evident that the control effort corresponding to the controller designed deterministically (design point G) is superior to that of the robust point obtained probabilistically (design point B) in this study. However, in order to compare the robustness behavior of the obtained optimum robust controller design (point B) of this robust study with that of the design (point G) obtained in deterministic design, two identical MCSs with 10000 sample generations are performed for each design. The values of means and variances of the control effort corresponding to each individual are computed using equations (4) and (5) are given in Table 3. Clearly, the superiority of the robust performance of the design point B to that of design point G can be observed from this table. Both the values of mean and, more importantly, the variance of the control effort of design point B are outstandingly less than those of the design point G. Such robustness can also be observed in Figure 10 where the cumulative distribution functions (CDF) of the statistical performances of both design points B and G obtained by MCSs are shown. It is also very evident from this figure that the probability of high control effort (for example CE value with more than 37296) of design point B is only 10 (1-0.9=0.1) percent whilst such value of the probability for the design point G of the certain design stands for values of CE more than 192074

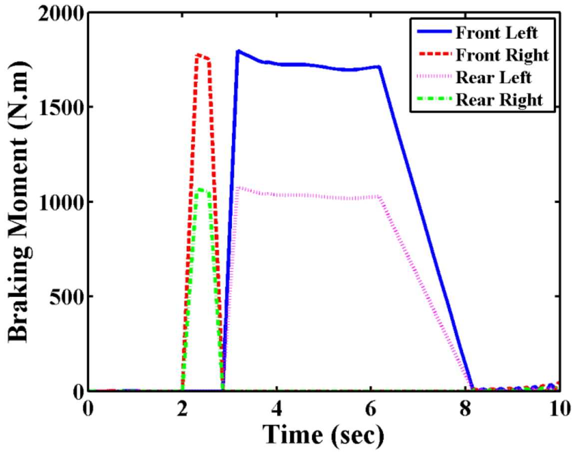


Fig6. Control input of the optimal point B of the Pareto front shown in Fig. 3.

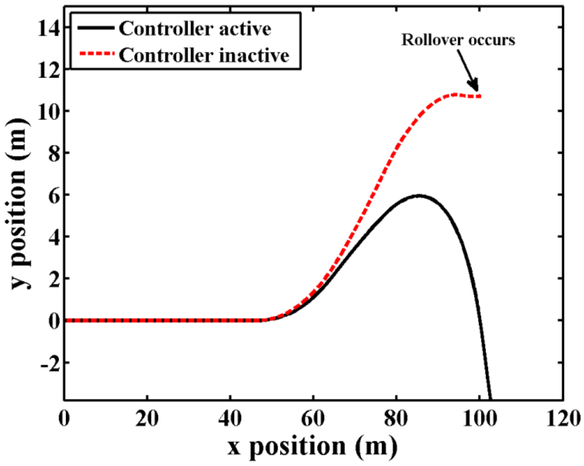


Fig7. Vehicle trajectories corresponding to the optimum design point B of the Pareto front shown in Fig. 3 during the fixed fishhook maneuver.

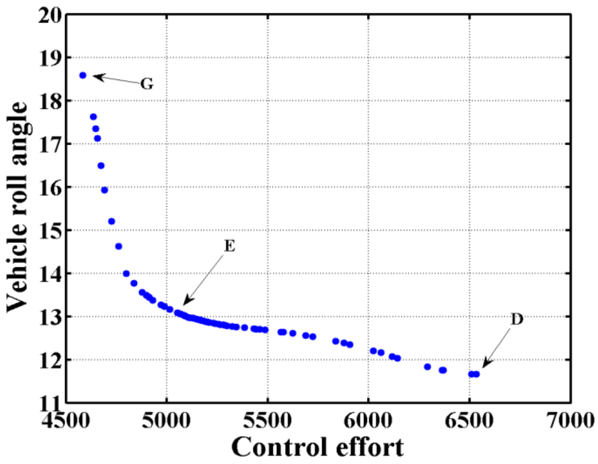


Fig8. The obtained Pareto front using the MUGA for deterministic optimal controller design.

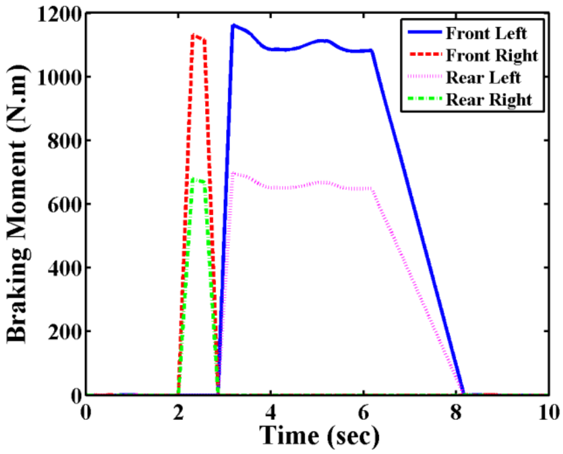


Fig9. Control input of the optimal point G of the Pareto front shown in Fig. 8.

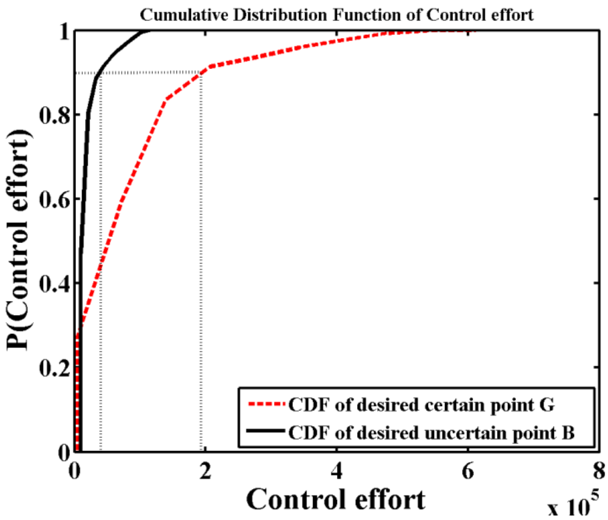


Fig10. CDFs of the optimum robust point B and of deterministic optimum design point G by MCS (10000 samples).

6. Conclusion

A multi-objective uniform-diversity genetic algorithm (MUGA) was successfully used to optimally robust sliding mode controller (SMC) design for vehicle rollover prevention. The objective functions that conflict with each other were appropriately selected as the mean of control effort (MCE) and variance of control effort (VCE). The multi-objective optimization of the robust mechanism led to the discovering some important trade-offs among those objective functions. The robustness of the design point obtained by such hybrid application of MUGA and MCS to the optimal controller design has been appropriately shown using the statistical metrics of mean and variance. The framework of such hybrid application of multi-objective GAs and MCS of this work for the Pareto optimum robust approach using some non-commensurable statistic objective functions is very promising and can be generally used in the optimal controller design with probabilistic uncertainty in order to vehicle rollover prevention.

References

[1]. National Highway Traffic Safety Administration (NHTSA), Motor Vehicle traffic Crash injury and fatality estimates, 2002 early assessment, NCSA (National Center of Statistics and Analysis) Advanced Research and Analysis, 2003.

[2]. Abe, M., Kano, Y., Suzuki, K., Shibahata, Y. and Furukawa, Y. (2001). Side-slip control to stabilize vehicle lateral motion by direct yaw moment. JSAE Review, 22, 413−419.

[3]. Bouton, N., Lenain, R., Thuilot, B., Fauroux, JC., A rollover indicator based on the prediction of the load transfer in presence of sliding: application to an All-Terrain Vehicle. IEEE International conference on robotics and automation roma , Italy, 10-14 April 2007

[4]. Yim, S., Design of a robust controller for rollover prevention with active suspension and differential braking , Springer, Journal of mechanical science and technology Vol. 26 (1) , (2012) 213-22

[5]. Kim, H., Robust roll motion control of a vehicle using integrated control strategy, Elsevier Control Engineering Practice 19 (2011) 820-827.

[6]. Lica, M., Cervantes, I., Robust Switched Predictive Braking Control for Rollover Prevention in Wheeled Vehicles, Mathematical Problems in Engineering Volume 2014, Article ID 356250.

[7]. Yim, S., Jeon, K., Yi, K., An Investigation into Vehicle Rollover Prevention by Coordinated Control of Active Anti-roll Bar and Electronic Stability Program, International Journal of Control, Automation and Systems (2012) 10(2) 275-287.

[8]. Kang, J., Yoo, J., Yi, K., Driving Control Algorithm for Maneuverability, Lateral Stability, and Rollover Prevention of 4WD Electric Vehicles With Independently Driven Front and Rear Wheels, IEEE Transactions on vehicular technology, vol 60, no. 7, sep 2011.

[9]. Imine, H., Fridman, L. M., Madani, T., Steering Control for Rollover Avoidance of Heavy Vehicles, IEEE Transactions on vehicular technology , vol. 61, NO. 8, OCTOBER 2012.

[10]. Sarel F. van der Westhuizen, Pieter S. Els , Slow active suspension control for rollover prevention, Elsevier (2013) 29-36.

[11]. Yim, S., Park, Y., Design of rollover Prevention Controller with linear matrix

inequality based trajectory sensitivity minimization, Vehicle System Dynamics, Vol. 49, No. 8, pp. 1225-1244, 2011.

[12]. Lim, D., Ong, Y.s., Lee, B.S., “Inverse multi-objective robust evolutionary design optimization in the presence of uncertainty”, GECCO’ 05, Washington, USA, pp. 55-62, 2005.

[13]. Papadrakakis, M., Lagaros, N. D., and Plevris, V., “Structural optimization considering the probabilistic system response”, Theoretical Applied Mechanics, Vol. 31, No. 3-4, pp. 361-393, Belgrade, 2004.

[14]. Wang, Q., Stengel, R.F., “Searching for Robust Minimal-order Compensators”, Journal of Dynamic Systems, Measurement, and Control, Vol. 123, pp. 223-236, June 2001.

[15]. Kalos, M.H., Whitlock, P.A., “Monte Carlo Methods”, Wiley, New York, 1986.

[16]. Wang, Q., Stengel, R.F., “Robust control of nonlinear systems with parametric uncertainty”, Automatica, Vol. 38, pp. 1591 − 1599, 2002.

[17]. Felezi, M., Reza pour, J., Nariman-Zadeh, N., Jamali, A., Fakhraei, M., Pareto optimal robust synthesis of a four-bar mechanisms with probabilistic uncertainties using a multi-objective uniform-diversity genetic algorithm(MUGA), International Symposium on Innovations in Intelligent Systems and Applications(INISTA) 21-24 June 2010, Kayseri & Cappadocia, TURKEY.

[18]. Crespo, L.G. and Kenny, S.P., “Robust Control Deign for systems with probabilistic Uncertainty”, NASA report, TP-2005-213531, March, 2005.

[19]. Pacejka, H. B. (2002). "Tyre and Vehicle Dynamics". Butterworth Heinemann.

[20]. Unsal, C., Kachroo, P., 1999, Sliding mode measurement feedback control for antilock braking systems, IEEE Transaction on control system technology 7 (2), 271-281.

[21]. Mirzaei, M., Mirzaei nejad, H., 2012. Optimal design of a non-linear controller for anti-lock braking system, Elsevier, Transportation Research Part C, 24, 19-35.

[22]. Pacejka, H. B., Besselink, I. J. M., Magic Formula tyre model with transient property, supplement to vehicle system dynamics, Vol 22, pp. 234-269, 1997.

[23]. Slotine, J.J., Li, W., "Applied nonlinear control", Prentice Hall, 1991.

[24]. Rezapour, J., Sharifi, M., Nariman-Zadeh, N., (2011) ”Application of fuzzy sliding mode control to robotic manipulator using multi-objective genetic algorithm", in publication of IEEE conference on International Symposium on Innovations in Intelligent Systems and Applications(INISTA), pp. 455-459.

[25]. Nariman-Zadeh, N., Saleh pour, M., Jamali, A., and Haghgoo, E., Pareto optimization of a five-degree of freedom vehicle vibration model using a multi-objective uniform-diversity genetic algorithm (MUGA), Engineering Applications of Artificial Intelligence, Vol.23 , pp. 543-551, 2010.

[26]. National Highway Traffic Safety Administration, Testing the dynamic rollover resistance of two 15-passenger vans with multiple load configurations, US Department of Transportation (2004).