Enhancing Rollover Threshold of an Elliptical Container Based on Binary-coded Genetic Algorithm

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Abstract

In this paper, a method based on binary-coded genetic algorithm is proposed to explore an optimization method, for obtaining an optimal elliptical tank. This optimization method enhances the rollover threshold of a tank vehicle, especially under partial filling conditions. Minimizing the overturning moment imposed on the vehicle due to c.g. height of the liquid load, lateral acceleration and cargo load shift are properly applied. In the process, the width and height of tanker are assumed as constant parameters. Additionally, considering the constant cross-sectional area, an optimum elliptical tanker of each filling condition is presented to provide more roll stability. Moreover, the magnitudes of lateral and vertical translation of the cargo within the proposed optimal cross section under a constant lateral acceleration field are compared with those of conventional elliptical tank to demonstrate the performance potentials of the optimal shapes. Comparing the vehicle rollover threshold of proposed optimal tank with that of currently used elliptical and circular tank reveals that the optimal tank is improved approximately 18% higher than conventional one.

Keywords: Rollover Threshold, Genetic Algorithm, Elliptic Tank, Optimization.

1. Introduction

Generally, Rollover accidents are one of the most common types of accident that occur in commercial vehicles carrying fluid. Since most of the time, these tankers are carrying dangerous liquid contents such as ammonia, gasoline and fuel oils, therefore stability of partially-filled liquid cargo vehicles is so important [1]. Therefore, in order to design the tank cross-sections, all parameters influencing rollover including sloshing frequency, sloshing forces, overturning moments and fluid c.g. height must be considered. The forces and moments arising from steering and braking maneuvers can lead to shift in roll and pitch planes due to sloshing of the liquid cargo [2]. Moreover, the additional overturning moment caused by dynamic load shift, affects the directional stability of the vehicle [3]. The most important factors influencing magnitude of load, (i.e. liquid force and overturning moment) are geometrical shape of the container, level and character of input and fill level of the liquid [4]. The roll stability of a partially filled vehicle is influenced by both the c.g. height and the magnitude of lateral load transfer. Different cross-section has been analyzed for optimizing roll stability.

The circular cross-section has a high center of mass location, but considerably less lateral load transfer with low fill volumes under a steady turning lateral acceleration field. On the other hand, modified oval tanks in comparison to circular tanks yield lower c.g. height and relatively larger lateral load transfer with low fill volumes. As a result the modified oval tanks exhibit higher roll stability limits than circular tanks [4-5]. Semi-rectangular tanks and modified oval tanks achieve lower fluid c.g. height by varying the top and the side radius [6]. In order to achieve a lower c.g. height, a modified rectangular tank is proposed by popov et al. investigated the optimal shape of rectangular and elliptical containers based on numerical analyzes of liquid loads [7]. The application of GA's method in predicting rollover of commercial vehicles carrying liquid cargo has been developed in recent years. Genetic algorithm changes tank cross-section for minimizing overturning moment which optimizes roll stability of generic tank vehicles [8]. However, this method is not used in optimizing elliptical tank.

In this paper, GA is used to optimize the rollover threshold of an elliptical tank and also to determine optimum cross-section of a particular type of cylindrical tank. In order to determine an optimal

cross-section with a proper lateral load transfer and also rollover threshold which gather both features of cylindrical and elliptical tanks, effective parameters such as overturning moments , fluid c.g. height, cross-sectional areas and dimensions of tankers are considered in genetic algorithm optimization. Consequently, determination of certain operators such as selection, cross over and mutation has been carried out to provide a proper design. With comparing the GA results and numerical methods, it has been deduced that the optimal elliptical cross-section proposed by the GA method is one of the best elliptical cross-sections which enhances tank roll stability.

Fluid Center of Gravity

Roll plane model of a tanker with assuming quasi-static fluid sloshing is used in this analyzes. Lateral acceleration and roll of the tanker causes motion of the fluid mass center [9]. In this analysis, rotation angle is assumed to be small and the fluid viscosity is negligible [10]. Equation (1) shows an elliptical tank shape.

$$\frac{x^2}{a^2} + \frac{(y-b)^2}{b^2} = 1 \tag{1}$$

where x and y are the coordinates of elliptical geometry, 2a and 2b are elliptical height and width respectively, equation (1) can be rewritten as:

$$y = b\left(1 \pm \sqrt{1 - \left(\frac{x}{a}\right)^2}\right) \tag{1}$$

Equation of the free surface of fluid without sprung mass, roll angle and lateral acceleration represented as:

$$y = -h_0^{(i)}, \quad i = 1,...,n \tag{2}$$

h_0 indicates height of fluid free surface. While equations (1-3) are solved simultaneously, the intersecting points of the fluid free surface containing (x_p, y_p) and (x_q, y_q) are obtained. With comparing $h_0^{(i)}$ with the maximum and minimum values of the arcs limits, cross-sectional area are obtained. As (x_p, y_p) and (x_q, y_q) are symmetrical points with respect to y axis in the absence of lateral acceleration and sprung mass roll angle, therefore $x_p = -x_q$. The volume of fluid per unit of the tanker length is predicted as follows:

$$A_0^{(i)} = 2\int_0^{y_p} \int_{f_1(x)}^{h_0(i)} dx\,dy \tag{3}$$

$f_1(x)$ expresses the equation of tank geometry. Equation of the free surface of fluid under steady turning can be presented as:

$$y = -\left(\frac{\theta_{s3} - a_y}{1 + \theta_{s3} a_y}\right)x + \kappa^{(i)} \quad, i = 1,...,n \tag{4}$$

θ_{s3} is the roll angle of sprung mass, a_y is the lateral acceleration in g unit and $\kappa^{(i)}$ is intersection of the y-axis with the free surface of the fluid. The total volume of fluid per length unit is compared to the initial volume, in the case that $a_y = 0$ and $\theta_{s3} = 0$. The error values corresponding to fluid volume are estimated as:

$$\varepsilon = |A^{(i)} - A_0^{(i)}| \tag{5}$$

The process would be repeated again until the error function rate goes beyond the allowable error. Therefore, coordinates of fluid c.g. are represented in the equation (7):

$$\begin{cases} \overline{Y_l}^{(i)} = \dfrac{1}{A^{(i)}} \int_{y_p}^{y_q} \int_{f_1(i)}^{f_2^{(i)}} y\,dy\,dx; i = 1, 2,...,10 \\ \overline{X_l}^{(i)} = \dfrac{1}{A^{(i)}} \int_{y_p}^{y_q} \int_{f_1(i)}^{f_2^{(i)}} x\,dy\,dx; i = 1, 2,...,10 \end{cases} \tag{6}$$

For one iteration if $b \le h \le 2b$, cross-sectional area (A_h) and fluid c. g. height (y_{CG}) are represented as:

$$A_h = \pi ab - \alpha \tag{7}$$

Where α is written as:

$$\alpha = 2\left\{ x_h b - x_h h + \frac{b}{2a}\left[x_h\sqrt{a^2 - x_h^2} + a^2 \sin^{-1}\left(\frac{x_h}{a}\right) \right] \right\} \tag{9}$$

$$y_{CG} = \frac{\pi ab^2 - \alpha\beta}{A_h} \tag{10}$$

While

$$\beta = 2b - y_{CG} \qquad (11)$$

Overturning moment caused vertical and lateral movement of fluid c.g. represents rollover potential in tank vehicle [11]. When a partially filled tank is subjected to a lateral acceleration, the free surface of fluid will gain slope of φ. This slope numerically equals to the lateral acceleration and is expressed to the gravity term, g_x. With the aid of graphic software, the fluid free surface gradient under lateral acceleration, tangents to an ellipse with a ratio of length to width of the tanker. In addition, the fluid c.g. move on an elliptical path with the same aspect ratio [12]. Moreover, the elliptical curve shapes due to the path of c.g. motion, tangent of the free surface of liquid have the same centers as the elliptical has.

Coordinates of x_l and y_l which are fluid center of gravity are shown as follows:

$$x_l = b_l \cos \varphi, \quad y_l = b_l \sin \varphi \qquad (12)$$

b_l Is known from the fluid c.g. position (Figure 2). The slope of fluid in an elliptical container is represented as:

$$\frac{dy_l}{dx_l} = -\left(\frac{a_l}{b_l}\right)\left(\frac{1}{\tan \varphi}\right) = w = g_x \qquad (13)$$

With replacing r equals to the ratio of the elliptical diameters:

$$\frac{a_l}{b_l} = \frac{a_r}{b_r} = \frac{a_h}{b_h} = r \qquad (14)$$

The equation (13) is formulated as:

$$w = g_x = -r\left(\frac{1}{\tan \varphi}\right), \quad \varphi = \tan^{-1}(\frac{-r}{g_x}) \qquad (15)$$

The location of the fluid center of gravity is represented as:

$$x_l = b_h \cos \varphi, \quad y_l = b_h \sin \varphi \qquad (16)$$

$$x_l = b_h \cos\left(\tan^{-1}(\frac{-r}{g_x})\right), \quad y_l = b_h \sin\left(\tan^{-1}(\frac{-r}{g_x})\right) \qquad (17)$$

b_h, a_h and other symbols are known from the fluid c.g. position as shown in Figure (2). Equation (18) represents overturning moment around point O, the bottom point in the tanker, due to fluid shifts and lateral acceleration.

$$M_z = ma_x\left(b - a_h \sin\left(\tan^{-1}(\frac{-r}{g_x})\right)\right) + mg\left(b_h \cos\left(\tan^{-1}(\frac{-r}{g_x})\right)\right) \qquad (18)$$

The equation (18) represents the overturning moment as a function of fluid c.g. height and also the geometry parameters.

2. Rollover Threshold

Rollover threshold of partially filled tanker is estimated by overturning moment and restoring moment balancing, assuming that the roll angle of unsprung mass compared to sprung mass is negligible. The model of a vehicle without suspension system is shown in Figure 1. Initial overturning moment caused by lateral acceleration and force shifting is shown in equation (19).

$$M = -\left(m_s h_{cs} + w_l h_l\right)a_x \qquad (19)$$

Where h_{cs} is sprung mass height, h_l height of fluid c.g., m_s vehicle weight and w_l fluid weight. Fluid c.g. shift in partially filled tank imposes additional overturning moment, which is expressed in the following equation.
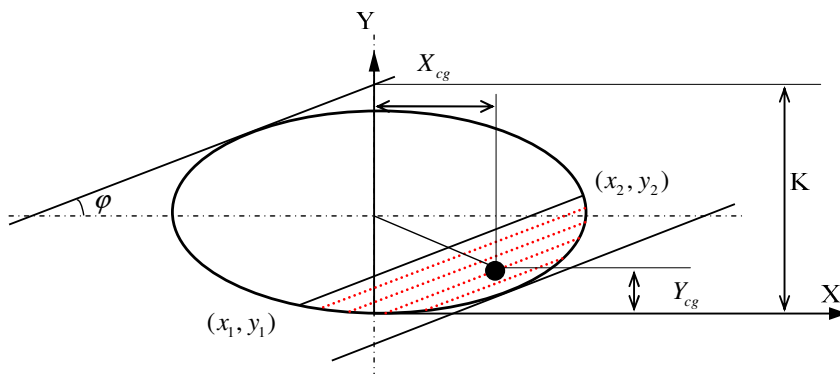

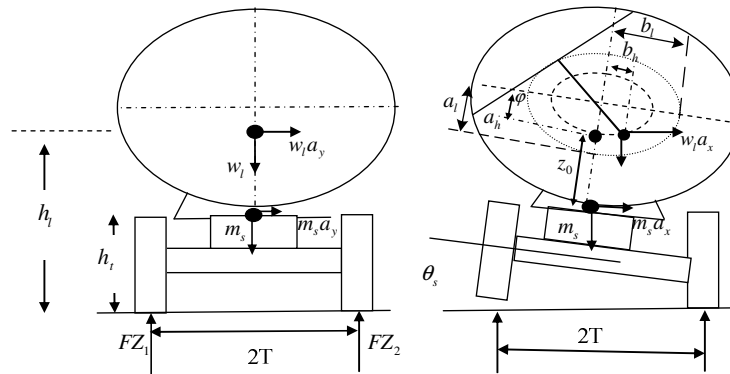**Fig1.** Elliptical parameters under steady lateral acceleration

Fig2. roll plane model of tank vehicle

Table1. Vehicle specification in roll plane model [3]

Vehicle Characteristics	Value
Sprung mass, m_s	5000 kg
Dual tire damping, c_t	2.5 kNs/m
Half-track width, l_w	1.0 m
Half spring spacing, l_u	0.7 m
Sprung mass c.g. height, h_{cs}.	1.5 m
Axle roll center height, h_{ra}	0.4 m
Turning radius, R	250 m

$$M' = -w_l X_l \tag{20}$$

Assuming that θ_s is a small, φ equals to a_y and for cylindrical tank R is radius. The equation (20) is rewritten as:

$$M' = -w_l(R - Z_0)a_y \tag{21}$$

Z_0 is c.g. height from the tank bottom. The moment due to lateral displacement of the vehicle under the influence of sprung mass roll (θ_s) is presented in equation (22).

$$M_2 = -(m_s h_t + w_l h_l)\theta_s \tag{22}$$

Inner and outer vehicle tire force caused restoring moment. The restoring moment is written as:

$$M_3 = (FZ_2 - FZ_1)T \tag{23}$$

FZ1 and FZ2 are left and right reaction forces of tires. When the value of M_3 is larger than M_2, the vehicle is unstable and maximum value of overturning moment equals to $(m_s + w_l)T$ (the total amount of load transfer) and moment of M_2 can be removed. Rollover threshold of solid state and completely filled mode is represented as a_y and partially filled mode a_{y1} Y_l and X_l are obtained from balance equations. In the case of rigid or fully loaded tanker the corresponding acceleration is presented as:

$$a_y = \frac{m_s + w_l}{m_s h_t + w_l h_l}T \tag{24}$$

In the case of partially filled circular tanks the following equation is deduced as [13]:

$$a_{y1} = \frac{m_s + w_l}{m_s h_t + w_l h_l + w_l(R - z_0)} T \qquad (25)$$

Rollover threshold of MC307 cylindrical tank with the diameter of 2.03 (m) is shown in Figure 3.

Rollover threshold of an elliptical container assuming four degrees of freedom suspension in the roll plane model is represented as:

$$a_{y1} = \frac{m_s + m_{us} + w_l}{m_s h_{cs} + m_{us} h_{ca} + w_l h_l + w_l(a - \frac{a}{b} z_0)} T \qquad (26)$$

m_{us} is unsprung mass and h_{ca} is unsprung mass height. In the case that, the tanker carrying solid cargo, the terms of load shifting in equation (26) will be removed. Thus, (27) equation will be obtained.

$$a_{y1} = \frac{m_s + m_{us} + w_l}{m_s h_{cs} + m_{us} h_{ca} + w_l h_l} T \qquad (27)$$

Figure 4 shows rollover threshold of a roll plane model in a partially filled tanker. The corresponding height of the container is constant while its width could be increased. As depicted in Figure 4, with increasing the width, stability of the tanker is considerably reduced, due to increasing the overturning moment.

The solid and partially filled liquid cargo is compared in Figure 5 where the aspect ratio (a/b) is assumed to be 2. As depicted, the solid cargo shows more stability than partially filled one due to sloshing effects.
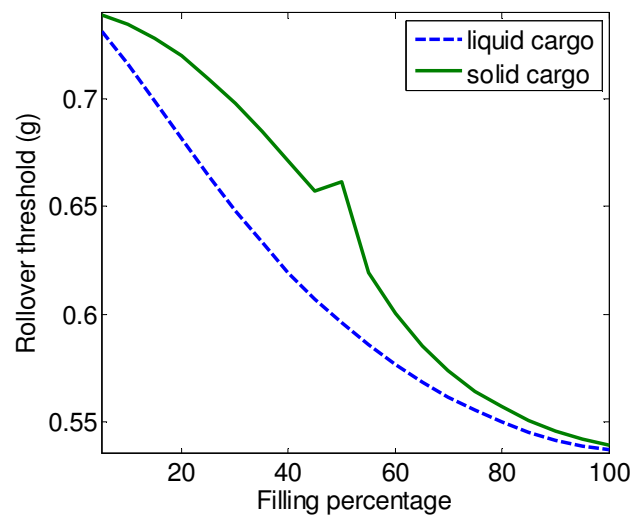

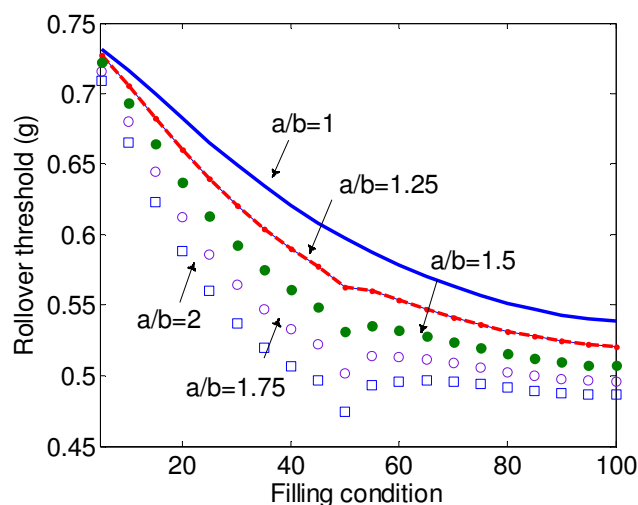**Fig3.**Overturning moment of solid and liquid cargo

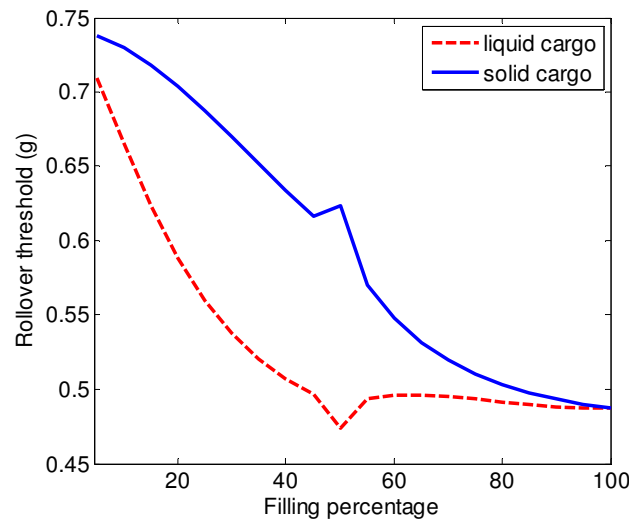**Fig4.**Rollover threshold of elliptical containers

Fig5. Rollover threshold of elliptical tanker in solid and liquid cargo
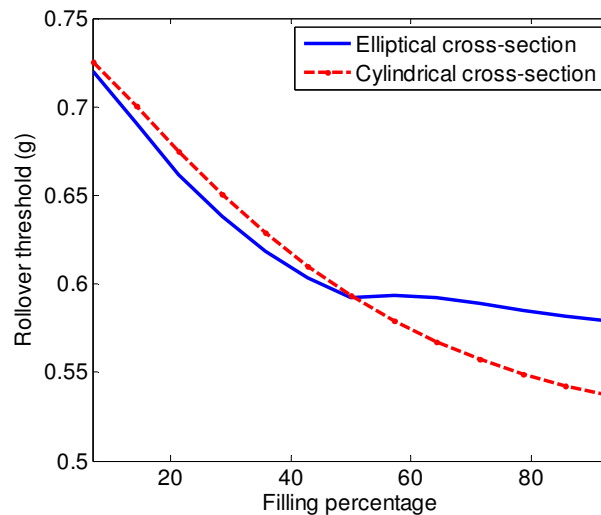

Fig6. Rollover threshold of elliptical and circular cross sections

Contrary to expectations, the elliptical tank is more stable than the cylindrical one. As represented in Figure 6, below 50% filling, cylindrical tank is more stable. This is because of the contrast between fluid c.g. height and overturning moment. As the filling level enhances, liquid cargo shows more similar behavior of solid cargo, this is due to the fact that the fluid c.g. height plays more important role than overturning moment

3. Optimization Process

Nowadays using evolution methods to solve optimization problems has a growing trend. Genetic algorithms as the most popular evolution algorithms have a extensive application in search studies [14-15]. Genetic algorithm begins with a number of solutions

called population. These solutions are represented by strings of gene chromosomes. They are taken from a population of solutions and are used to create new populations, with this thought that the new population is better than the original population. Mutation operator changes the integer parts of strings. Mutation of each string is proportional to the initial threshold value, which is chosen to fit the mutation rate. This operator can provide a solution that does not exist in the population to compensate loss of valuable information resulting from inadequate intersection.

The two objective functions that are optimized in this paper contain overturning moment and fluid c.g. height. These functions attempt to be minimized in the optimization process to produce the most proper elliptical area. Area constraints and also constraints applied to the variables in equation (18) play an

important roll in minimizing the functions. The elliptical curves are shaped using five variables applied as control points. These variables are optimized during the optimization process. This optimization process is based on the sum of weighted cost functions.

$$U(X) = Minimize \ [w_1 M_0 + w_2 Y_l], \quad w_1 = w_2 = 0.5 \tag{28}$$

These, weighting coefficients are selected based on cost function importance. In this paper their values

are equal to 0.5.

Figure 9 represents a summary of the GA performance in the function through optimization process. Randomly generated individuals in the population indicate overturning moment and fluid c.g. height. The area constraint is defined to reduce the number of iterations in the generation process and prevent generating inappropriate individuals in the population.
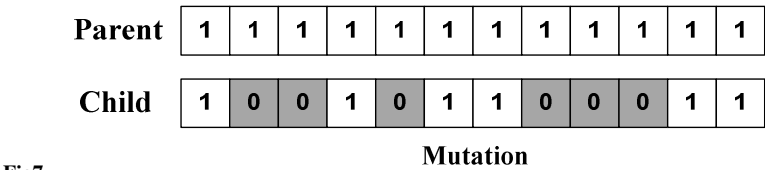

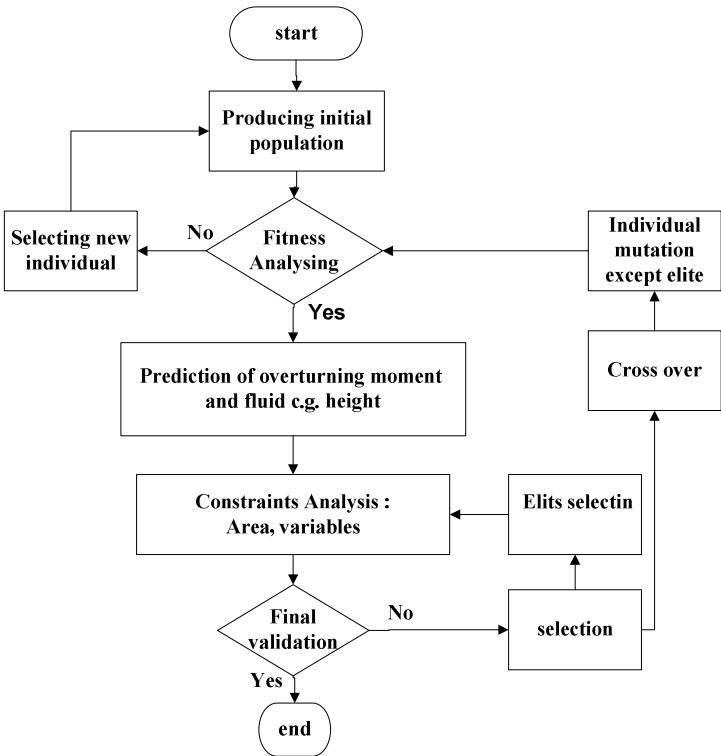**Fig7.**

Fig8. Genetic algorithm operator

Fig9. Genetic algorithm diagram

4. Numerical Results

GA algorithm in Figure 9 includes a large number of equations and different parameters such as iterations, maximum cost function, stopping criteria and population number. Getting the lowest iteration

would be important to achieve the desired response. An unsuitable value for the mutation rate causes premature convergence and therefore does not get the best optimization response. To investigate the effect of mutation rate, a cylindrical tanker with total area of 3.256 m2 is considered for optimization [5]. Mutation rate according to Figure 10 is selected equals to 0, 3,

6 and 9. Without mutation rate, premature convergence occurs and the genetic algorithm will not be able to analyse the best solution of the problem. Considering some random points in the population, may be desirable to find the better choice in optimization process. As depicted in Figure 10 where the zero mutation rates is chosen, the program is rapidly convergent and the solution is obtained with 60 iterations while the overturning moment is 1866 N/m. When the mutation rate is chosen 3%, the solution is obtained in 90 iteration while the overturning moment is 1900 N/m. With imposing the mutation rate, the chances of new individuals and also the number of generation are enhanced. As it is anticipated, the mutation rate prevents local convergence. Moreover average fitness values in each generation would be increased with decreasing the mutation rate. As shown in Figure 10 as mutation rate increases, the convergence would be reduces and algorithm does not lead into a proper cost value. It can be observed that the best mutation rate in the present optimization process is chosen the value between 4-6%

Further exploration is performed to study the effect of proposed GA by comparing optimization results with numerical analysis. To analyze the genetic algorithms output, overturning moment and rollover threshold of the optimized elliptical tanker are compared with those of cylindrical tanks and also conventional elliptical tanks (Figure 11-13).

Accuracy of GA method for determining the optimal cross-sectional area with the lowest overturning moment is compared with the conventional method based on numerical analysis (Figure 11). As shown in Figure 11, overturning moment of optimal cross-section is less than circular and conventional elliptical tanker. As observed, the circular tank vehicles does not present proper overturning moments. The best results belong to present optimized tank in which the corresponding overturning moments reduced considerably. In other hand, 23% reduction is observed comparing the corresponding overturning moment with the conventional elliptical tank.
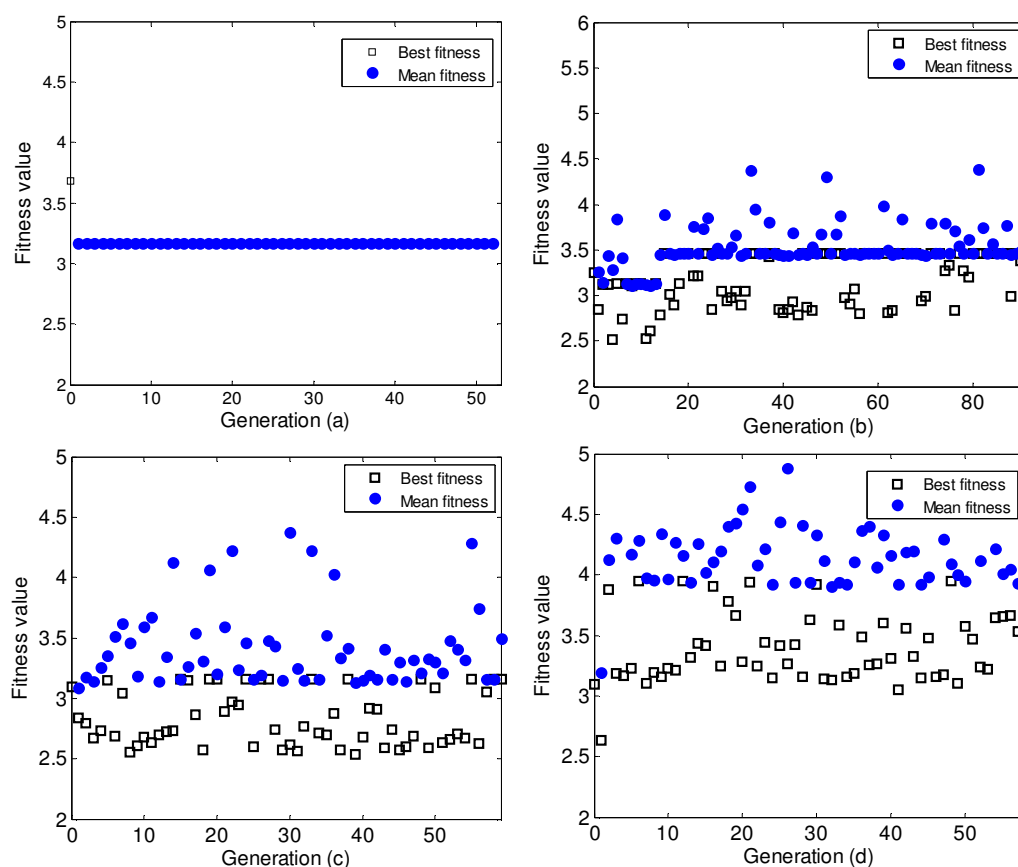

Fig10.   Comparing Convergence of fitness function in (a) 0% mutation rate, (b) 3%, (c) 6% and (d) 9%
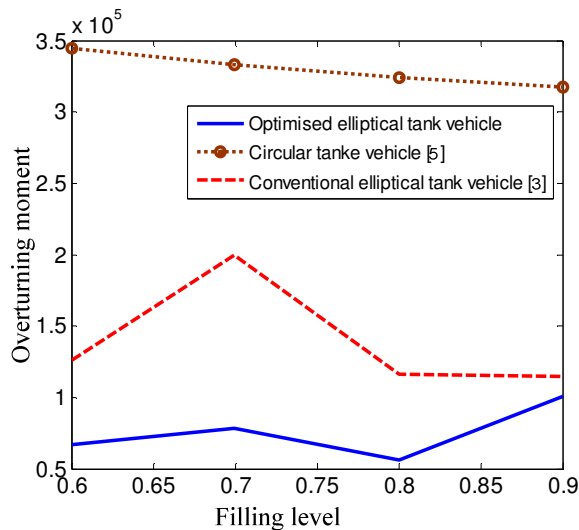
Fig11.   Comparing overturning moment of optimal elliptical cross-section and conventional elliptical cross-section
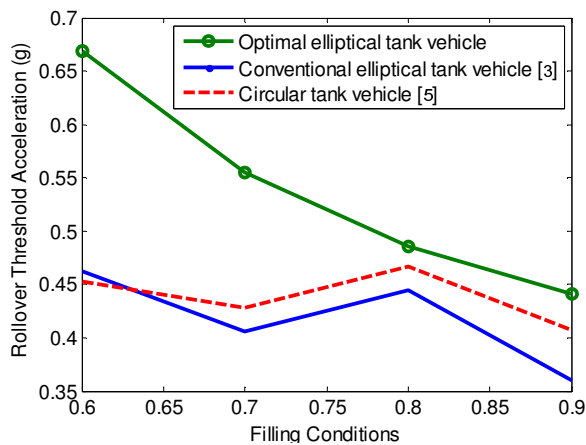

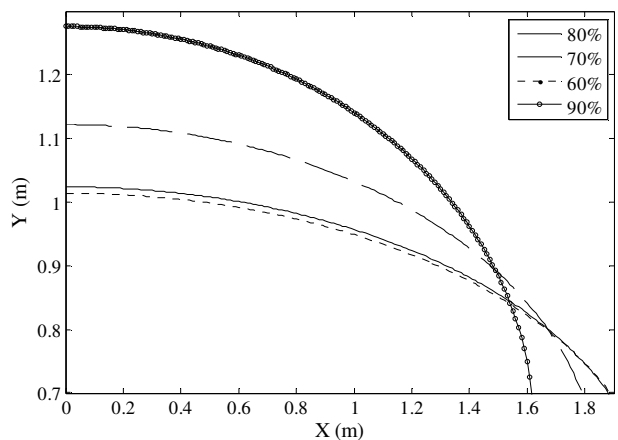**Fig12.**   Rollover threshold of elliptical and circular container

Fig13.   Optimized elliptical cross-sections in different filling conditions

In addition, the rollover threshold is enhanced in optimized elliptical tank in comparison to circular and conventional elliptical tanks as depicted in Figure 12. The figure also indicates that the rollover threshold in lower filling condition is more dependent to optimum cross-section due to lower c.g. height. The results also confirm that in higher filling condition the effects of optimized cross-section on rollover threshold are considerably reduced, although, the results still have revealed the best condition. The results indicate that the proposed optimized elliptical shape enhances rollover threshold of elliptical tank approximately 18% in comparison to conventional elliptical tanks.

The optimal elliptical cross-section is plotted for different filling conditions and is shown in Figure 13. These optimal shapes are concluded by minimizing overturning moment and fluid c.g. height. It should be noted that, each optimized cross-section gives the best optimized results in corresponding filling condition.

5. Conclusion

In this paper a method based on genetic algorithm was proposed to determine the optimized cross-section with the lowest overturning moment and fluid c.g. height. In this regard, a program was developed to predict the optimized shape based on conventional elliptical tank. The results show that the proposed method is able to enhance tank roll stability with high accuracy. In addition, the genetic algorithm proposed in this research is extended to obtain the better and faster convergence. In the present optimization process, the mutation rate should be selected between 3 to 6 percent and individuals should be at least 40. Finally, the results presented in this paper indicate that the proposed optimized elliptical shape based on binary-coded GA method enhances rollover threshold of elliptical tank approximately 18% in comparison to conventional elliptical tanks. In addition, overturning moment of elliptical tank is reduced approximately 23% in comparison to conventional elliptical ones.

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