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Nonlinear model identification of oil-lubricated tilting pad bearings

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ABSTRACT

A new method of modeling and identification of weak nonlinear behavior of tilting pad journal bearings (TPJBs) is proposed. A novel nonlinear model of a TPJB based on a Taylor series expansion of oil film forces, consistent with the transient form of Reynolds differential equation (the original model), is developed. The least mean square technique in time domain is employed to identify Taylor series coefficients of the nonlinear model. The terms with dominant effects on the nonlinear oil film forces are chosen using the subset selection method. Good agreement is achieved between the predicated response obtained from the original model and the one evaluated through using the identified nonlinear model.

1. Introduction

Nowadays, hydrodynamic tilting pad journal bearings (TPJB) are widely used in industrial rotating machines for their excellent stability, high radial load capacity and high speed operating conditions. Dynamical behavior of a journal bearing may be fully characterized by its operating conditions. These include journal angular velocity, applied radial load, inlet oil temperature/pressure, bearing's material and, its geometry. The Reynolds differential equation along with the heat equations which are incorporating the aforementioned characteristics, have been basically used to model the pressure and temperature distributions inside the journal bearing. A common approach for modeling the nonlinear oil film forces in plain journal bearings is based on Taylor series expansion of oil film forces by perturbed displacements and velocities from journal equilibrium position. In a specified rotor-bearing system, all the operational conditions and the geometric and material properties, except the journal angular velocity, are usually kept constant. Thus, in such a case, the dynamic coefficients evaluated by Taylor series expansion of oil film forces may become dependent only on the angular velocity. The speed dependent dynamic coefficients are often used as a standard representation of a foundation dynamic model in a given rotating machine.

The Taylor series expansion of oil film forces with the assumption of linear behavior of journal bearing dynamics will yield eight stiffness and damping coefficients. The dynamic modeling of journal bearings, specifically the TPJBs, by linear dynamic coefficients is a common approach to linear dynamic analysis of rotor-bearings systems, and researchers have proposed several procedures for evaluating such coefficients over past decades [1–8]. However, linear coefficients may not be used in an accurate prediction of unbalance response while rotating parts experiences large amplitude vibration due to high dynamic loads. Under these conditions, the nonlinear model of oil film forces in journal bearings must be employed in order to accurately predict the dynamic response of the rotating machines. Although there are several existing studies in the literature concerning the calculation of linear dynamic coefficients in different types of journal bearings such as TPJBs, the evaluation of nonlinear dynamic coefficients have been confined to plain journal bearings. Choy et al. [9] expanded the nonlinear oil film forces using the odd power series of journal displacement from its equilibrium position. They showed that at displacements far away from the equilibrium position, the higher order terms of nonlinear stiffness coefficients dominate accuracy of nonlinear oil-film force predictions. Czolczynski [10] described an identification procedure for the linear and nonlinear stiffness and damping coefficients of gas bearings. The procedure estimates dynamic coefficients of bearing response assuming it follows a simple harmonic motion. However, this assumption is not valid in general, as the journal response under high dynamic loads may contain higher harmonies which are not predicted by the adopted assumption. Chu et al. [11] described a quasi-static nonlinear dynamic model in order to include higher order terms in a bearing reaction expansion to capture nonlinearity within the oil film forces. They described and used an error evaluation scheme to set confidence bounds on the higher order results. It was shown that the oil film force nonlinearities may be

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Nomenclature

c_{ij}; $i,j=X,Y$ linear damping coefficients
$\overline{C}_P$ oil specific heat capacity
D_b bearing diameter
D_J journal diameter
e unbalance mass eccentricity
f_p fractional angular position of pivot
F_X, F_Y oil film forces in the x and y directions
F_X^{NL}, F_Y^{NL} nonlinear parts of the oil film forces in the x and y directions
h_k oil film thickness in the kth pad
H_{p1} heat convection coefficient between the inner pad surface and the oil film
H_{p2} heat convection coefficient between the outer pad surface and the oil film
H_{p3} heat convection coefficient between the leading/trailing edges of the pad and the oil film
H_J heat convection coefficient between the journal surface and the oil film
I_P pad moment of inertia in relation to pivot center
k_{ij}; $i,j=X,Y$ linear stiffness coefficients
K_{xi}, K_{yj} ith and jth nonlinear dynamic coefficients in x and y directions
k_{xi}, k_{yj} dimensionless ith and jth nonlinear dynamic coefficients in x and y directions, respectively
k_{p1} heat conduction of the inner surface of the pad
k_{p2} heat conduction of the outer surface and the leading/trailing edges of the pad
L axial bearing length
m preload
m_J journal mass
M_k moment applied to the kth pad
O_B center of the bearing
O_J center of the journal
P_k pressure distribution in the kth pad
P_{sup} oil supply pressure
Q_{sup} oil supply flow rate
t_p pad thickness
T_{sup} oil supply temperature
T^k temperature distribution in the oil film between the kth pad and the journal
T_{Sp}^k temperature distribution on the inner surface of the kth pad
T_J journal surface temperature
T_p^k temperature distribution in the kth pad
R_b bearing radius
R_J journal radius
R_p pad radius
$R_{op}=R_p+t_p$ outer pad radius
W_b radial static load on bearing
X,Y journal displacements in x and y directions
z bearing axial coordinate
δ_k tilt angle in the kth pad
Δ_p angular dimension of pad
θ bearing circumferential coordinate
θ_p pivoting angles of pads
θ_p^k pivoting angle of the kth pad
θ_l^k leading angle of the kth pad
θ_t^k trailing angle of the kth pad
μ dynamic oil viscosity
Ω angular velocity of the journal
ρ oil density
β thermo-viscosity index

significant and the linear model to be valid only within small amplitudes. The variations of stiffness and damping coefficients of bearing oil films along the locus of rotor center were obtained by means of numerical analysis of a rotor-journal bearing system exposed large dynamic loads [12]. Most of the dynamic coefficients vary by an order of magnitude. This wide variation shows the noticeable influence of nonlinearities on the system dynamic response. A procedure using multiple regression analysis was developed to identify the nonlinear dynamic coefficients [13]. The dynamic coefficients were evaluated along the locus of journal center based on the third order Taylor series expansion of the oil film forces with respect to perturbed amplitudes about equilibrium position. Variations of nonlinear dynamic coefficients with reference to the linear ones were presented to establish a criterion for using the linear coefficients in different operating conditions. In a similar study [14], the variation of nonlinear stiffness and damping coefficients along the journal orbit with respect to equilibrium position was investigated. It was shown that the degree of nonlinearity introduced into the nonlinear dynamic coefficients in the nonlinear analysis might be larger at low eccentricity ratios than at high eccentricity ratios. Zhao et al. [15] proposed three types of nonlinear force models based on retaining certain terms of Taylor series expansion of oil film forces in plain journal bearings. The dynamic coefficients associated with the models were identified through applying a least-mean squares method in time domain. They concluded that reliable linear coefficients can be obtained through the models. However, the nonlinear coefficients in the proposed models may be evaluated reliably only if the excitation amplitude is large enough. Meruane and Pascual [16] developed a framework to identify nonlinear dynamic coefficients of a plain journal bearing from the journal forced response under high dynamic loading. The nonlinear oil film force model was represented through a third order Taylor series expansion which neglected the coupling terms between displacements and velocities. Weimin et al. [17] presented a computational method which is independent on the journal orbit to predict bearing dynamic coefficients using partial derivative method. In their study, the variation of dynamic coefficients with eccentricity ratios and length-to-diameter ratios of a plain journal bearing model have been studied. They concluded that rise in these parameters leads to increase in dynamic coefficients, and the nonlinear effects in oil-film forces are significant at heavy static loads. Yang et al. [18] extended the partial derivative method to identify the linear and second-order nonlinear dynamic coefficients of finite-length bearings. Then, the identified model was used to show the nonlinear dynamic behavior in a simple rotor-bearing system. Those obtained nonlinear motions have been studied by the phase portrait, journal trajectory and Poincare maps. They concluded that the proposed nonlinear modeling of oil film forces can be employed in the nonlinear dynamic analysis of other types of oil-film bearings. The aforementioned research [9–18] have indicated the ability of the nonlinear models based on higher order Taylor series expansion of oil film forces to identify the nonlinear effects within the associated journal bearings.

There exists a lack of study in employing the nonlinear dynamic coefficients for modeling oil film forces within more complicated journal bearings such as TPJB, which has $n+2$ DOFs where n is the number of pads and the other two DOFs are translational motions of the journal. A prevalent approach to model the nonlinearities in the TPJBs is solving the Reynolds equation for nonlinear forces and moments in the TPJBs using numerical integrations. Several researchers adopted this approach in order to evaluate the transient unbalance response of rotating systems supported on various TPJBs [19–30]. In a different study on transient behavior in TPJBs [31], an analytical model based on the short bearing theory has been proposed to calculate the nonlinear forces of a TPJB including the turbulence effect, and a comparison has been made between analytical models with and without this effect. In the study, the spatial integration of the Reynolds equation to obtain pressure distribution has been replaced by the closed form expression of the oil film forces, however the DOFs corresponding to pads still remains in the time integration process of the governing equations of motions. The approach based on numerical integration of the original governing equations of a TPJB may lead to higher computational accuracy compared to the approach which employs nonlinear dynamic coefficients; nevertheless, it takes longer time to have a reliable analysis by such a method.

In the present paper, a new methodology for modeling and identification of nonlinear dynamic coefficients in TPJBs is developed. The nonlinear model is characterized by expansion of oil film forces through Taylor series with nonlinear terms expressed using products of perturbed displacements and velocities. A novel arrangement of nonlinear dynamic coefficients, according to a basic understanding of the transient Reynolds equation, will be presented. The DOFs of the pads in the TPJB, incorporated in direct integration of the Reynolds equation (the original model), are eliminated in the proposed model. This order reduction makes the proposed nonlinear model a suitable alternative to the direct use of the original model for weak nonlinear transient analysis of TPJBs. The nonlinear dynamic coefficients are solely dependent on the journal angular velocity in a specified rotor-bearing system, and are identified using the oil film forces time series and journal orbits obtained through using the transient form of Reynolds differential equation (the original model). The numerically simulated data have been evaluated at five known levels of unbalance force, and, are used to setup an over-determined system of linear equations to find the nonlinear coefficients. A subset selection technique [32], is also used in order to choose the terms in the Taylor expansion, which are well representing the simulated oil film forces, among all candidate terms considered in the expansion.

The remainder of this paper is organized as follows. Section 2 illustrates the governing equations of a rigid rotor supported by a TPJB, within which the bearing is modeled by transient form of Reynolds differential equation (the original model). The approach adopted in the present study to model weak nonlinearities in a TPJB is presented, in Section 3. This approach leads to a reduced nonlinear model of a TPJB, which is originated from a basic understanding of the transient Reynolds equation. Section 4 describes the numerical simulation of dynamical behavior of a rigid rotor supported on a TPJB. The numerically simulated data are employed in Section 4 to identify the nonlinear dynamic coefficients of the reduced nonlinear model using subset selection technique. The proposed model is validated in this section by comparing its predicted response to the one obtained through applying the original model.

2. Governing equations

Rigid rotor equations of motion supported by a five pads TPJB, as displayed in Fig. 1, when subjected to an unbalance excitation is:

$$\begin{cases} m_J\ddot{X} = F_x + m_J e\Omega^2 \cos \Omega t \\ m_J\ddot{Y} = F_y + m_J e\Omega^2 \sin \Omega t - m_J g \\ I_P\ddot{\delta}_k = M_k\,, \quad k = 1, \ldots, N_{pads} \end{cases} \tag{1}$$

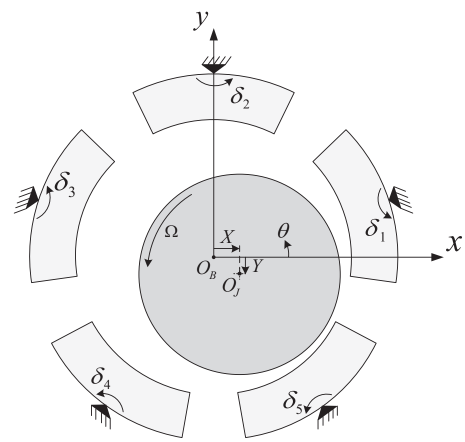

Fig. 1. Schematic representation of a TPJB with load-between-pads configuration.

The time transient two dimensional (2-D) Reynolds differential equation describing the pressure distribution in the kth pad is defined as:

$$\frac{1}{R_J{}^2}\frac{\partial}{\partial\theta}\left(\frac{h_k^3}{\mu}\frac{\partial P_k}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(\frac{h_k^3}{\mu}\frac{\partial P_k}{\partial z}\right) = 6\Omega\frac{\partial h_k}{\partial\theta} + 12\frac{\partial h_k}{\partial t};\ \theta_l^k \le \theta \le \theta_t^k\,,\quad 0 \le z \le L \tag{2}$$

where the oil film thickness in the kth pad is obtained using geometrical relations,

$$h_k = (R_p - R_J) - X\cos\theta - Y\sin\theta - (R_p - R_b)\cos\left(\theta - \theta_p^k\right) - \delta_k(R_p + t_p)\sin\left(\theta - \theta_p^k\right). \tag{3}$$

The one-dimensional (1D) pad temperature distribution is obtained from balance of energy expressed as:

$$\rho\overline{C}_P\frac{\Omega}{2}\frac{dT^k}{d\theta} = \mu\frac{(R_J\Omega)^2}{(h_k)^2} - \frac{H_{p1}\left(T^k - T_{Sp}^k\right) + H_J\left(T^k - T_J\right)}{h_k}. \tag{4}$$

To solve Eq. (4), one may employ the suggested recommendation of Ref. [33] as the initial condition for the temperature distribution:

$$T^k\left(\theta_l^k\right) = T_{\sup} + \frac{\min\left(\left\{h^j\left(\theta_l^j\right)\right\}_{j=1,\ldots,N_{pads}}\right)}{h^k\left(\theta_l^k\right)}\Delta T;\ k = 1, \ldots, N_{pads} \tag{5}$$

The pad temperature distribution is obtained by solution of two-dimensional (2D) heat equation along with its boundary conditions,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_p^k}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T_p^k}{\partial\theta^2} = 0;\ R_p \le r \le R_{op},\ \theta_l^k \le \theta \le \theta_t^k \tag{6-a}$$

$$B.C: \begin{cases} \frac{\partial T_p^k}{\partial r} = \frac{H_{p1}}{k_{p1}}\left(T_p^k - T^k\right);\ r = R_p \\ \frac{\partial T_p^k}{\partial r} = \frac{-H_{p2}}{k_{p2}}\left(T_p^k - T_{\sup}\right);\ r = R_{op} \\ \frac{1}{r}\frac{\partial T_p^k}{\partial\theta} = \frac{H_{p3}}{k_{p2}}\left(T_p^k - T_{\sup}\right);\ \theta = \theta_l^k \\ \frac{1}{r}\frac{\partial T_p^k}{\partial\theta} = \frac{-H_{p3}}{k_{p2}}\left(T_p^k - T_{\sup}\right);\ \theta = \theta_t^k \end{cases} \tag{6-b}$$

Assuming the shaft surface temperature to be circumferentially uniform, it is regarded as the average of oil film temperatures,

$$T_J = \frac{\sum_{k=1}^{N_{pads}} \int_{\theta_l^k}^{\theta_t^k} T^k d\theta}{N_{pads}\Delta_p} \tag{7}$$

The relationship between oil viscosity and its temperature is assumed to follow the exponential law given by:

$$\mu = \mu_0 exp(-\beta(T - T_0)) \tag{8}$$

Finally, the oil film bulk temperature rise is estimated from the followings:

$$\Delta T = \frac{\sum_{k=1}^{N_{pads}} R_J \Omega \int_{\theta_l^k}^{\theta_t^k} \mu \frac{R_J \Omega}{h_k} L R_J d\theta}{Q_{sup}\rho \overline{C}_P} \tag{9}$$

In this study, finite element and finite difference formulations are used simultaneously to perform the thermal analysis of the TPJB, seeking solutions to the 1D energy equation, Eq. (4), and the 2D heat equations, Eqs. ((6-a) and (6-b)), respectively. For a specified state of TPJB's DOFs and an initial guess for the oil film bulk temperature rise, the temperature distributions within the pads and the oil film can be evaluated and updated iteratively, through Eqs. (3)–(9). Subsequently, one may easily calculate viscosity variations in the oil film through applying the determined oil film temperature distributions to Eq. (8). Eventually, thermo-hydrodynamic (THD) analysis of the TPJB, considering the evaluated oil film viscosity variations, has been implemented in order to solve the Reynolds equation, Eq. (2), for the pressure distribution in the bearing. Finite element discretization of Eq. (2) is performed to evaluate the pressure distribution in the oil film.

Using the obtained pressure distribution, one can evaluate numerically the nonlinear oil film forces and moments as:

$$\begin{cases} F_x = -\sum_{k=1}^{N_{pads}} \int_0^L \int_{\theta_s^k}^{\theta_e^k} P_k \cos\theta R_J d\theta dz \\ F_y = -\sum_{k=1}^{N_{pads}} \int_0^L \int_{\theta_s^k}^{\theta_e^k} P_k \sin\theta R_J d\theta dz \\ M_k = -\int_0^L \int_{\theta_s^k}^{\theta_e^k} (R_J + t_p)(\theta - \theta_p) P_k \cos(\theta - \theta_p) R_J d\theta dz, \quad k = 1, \ldots, N_{pads} \end{cases} \tag{10}$$

In the present study, a reduced nonlinear model of TPJBs is proposed assuming:

1) Nonlinearity in the TPJB is weak.
2) The vibrating rotating system is over-damped.
3) Dominant exerted dynamic load on the rotating system is unbalance force excitation, synchronous with journal angular velocity.

Table 1
Properties of the TPJB under study.

Parameter	Value	Parameter	Value
f_p	0.5	L (mm)	50.000
D_J (mm)	125.000	W_b(N)	6880
D_b (mm)	125.196	I_p(kg m^2)	0.0024
θ_p(deg), LBP	18, 90, 162, 234, 306	m	0.3988
μ at 40 C (Pa s)	0.0396	Δ_p(deg)	52
$\overline{C}_P$(J/kg/K)	2000	P_{sup}(MPa)	0.15
ρ(kg/m^3)	860	Q_{sup}(l/min)	46.7
β(C^{-1})	0.0319	T_{sup}(C)	45
k_{p1}(W/m/K)	22	k_{p2}(W/m/K)	45

Under these assumptions, it may concluded that the free vibration response or any sub-synchronous vibrations, which incorporate frequencies different from the journal angular velocity or its integer multiples, do not contribute to the steady-state response with constant angular velocity, and may have negligible contribution to the transient response with constant angular accelerations. Consequently, journal angular velocity is the only harmonic component of the unbalance response in the linearized system, and it is the prime harmony of the response in presence of weak nonlinear behavior of the rotor supported by TPJBs. Therefore, synchronous linear dynamic coefficients, as linearized base model, are applied as leading dynamic terms to the model proposed for simulating weak nonlinearities in the TPJBs. Then, the reduced model by using nonlinear modifying terms added to TPJB linear synchronous dynamic coefficients, is expressed as:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ m_J g \end{Bmatrix} - \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} + \begin{Bmatrix} F_x^{NL} \\ F_y^{NL} \end{Bmatrix} \tag{11}$$

The next section, introduces a new nonlinear model which is consistent with the time transient form of Reynolds equation, Eq. (2).

3. Nonlinear modeling of TPJB

The nonlinear oil film forces in the TPJB, as stated in Eq. (10), are linearly dependent on the pressure distribution in the bearing, which is directly evaluated by finding a solution to the Reynolds equation. Thus, the relationship between the TPJB's DOFs and the pressure distribution holds between these DOFs and the oil film forces. The Reynolds equation is inherently a linear equation with respect to the pressure distribution. Therefore, a linear operator as:

$$L[] = \frac{1}{R_J^2}\frac{\partial}{\partial\theta}\left(\frac{h_k^3}{\mu}\frac{\partial[]}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(\frac{h_k^3}{\mu}\frac{\partial[]}{\partial S}\right), \tag{12}$$

can be defined to express Reynolds equation in the following form:

$$L[P_k] = 6\Omega\frac{\partial h_k}{\partial\theta} + 12\frac{\partial h_k}{\partial t};\ \theta_l^k \le \theta \le \theta_t^k,\ 0 \le z \le L \tag{13}$$

A direct solution to Eq. (13) for the pressure distribution is:

$$P_k = L^{-1}\left[6\Omega\frac{\partial h_k}{\partial\theta} + 12\frac{\partial h_k}{\partial t}\right];\ \theta_l^k \le \theta \le \theta_t^k,\ 0 \le z \le L \tag{14}$$

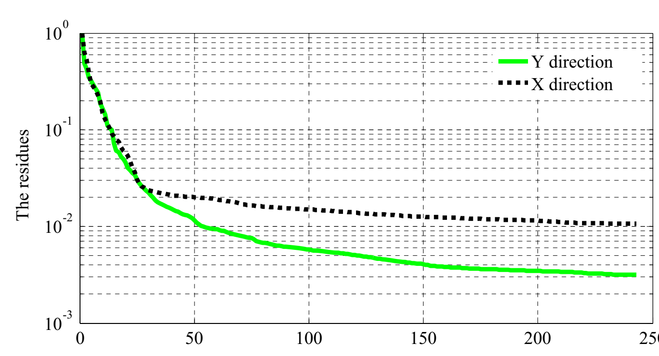

Fig. 2. The display of residues resulted from an application of subset selection, at Ω=5500 rpm and n=13, with respect to number of terms contributed in the nonlinear model, in a descending order.

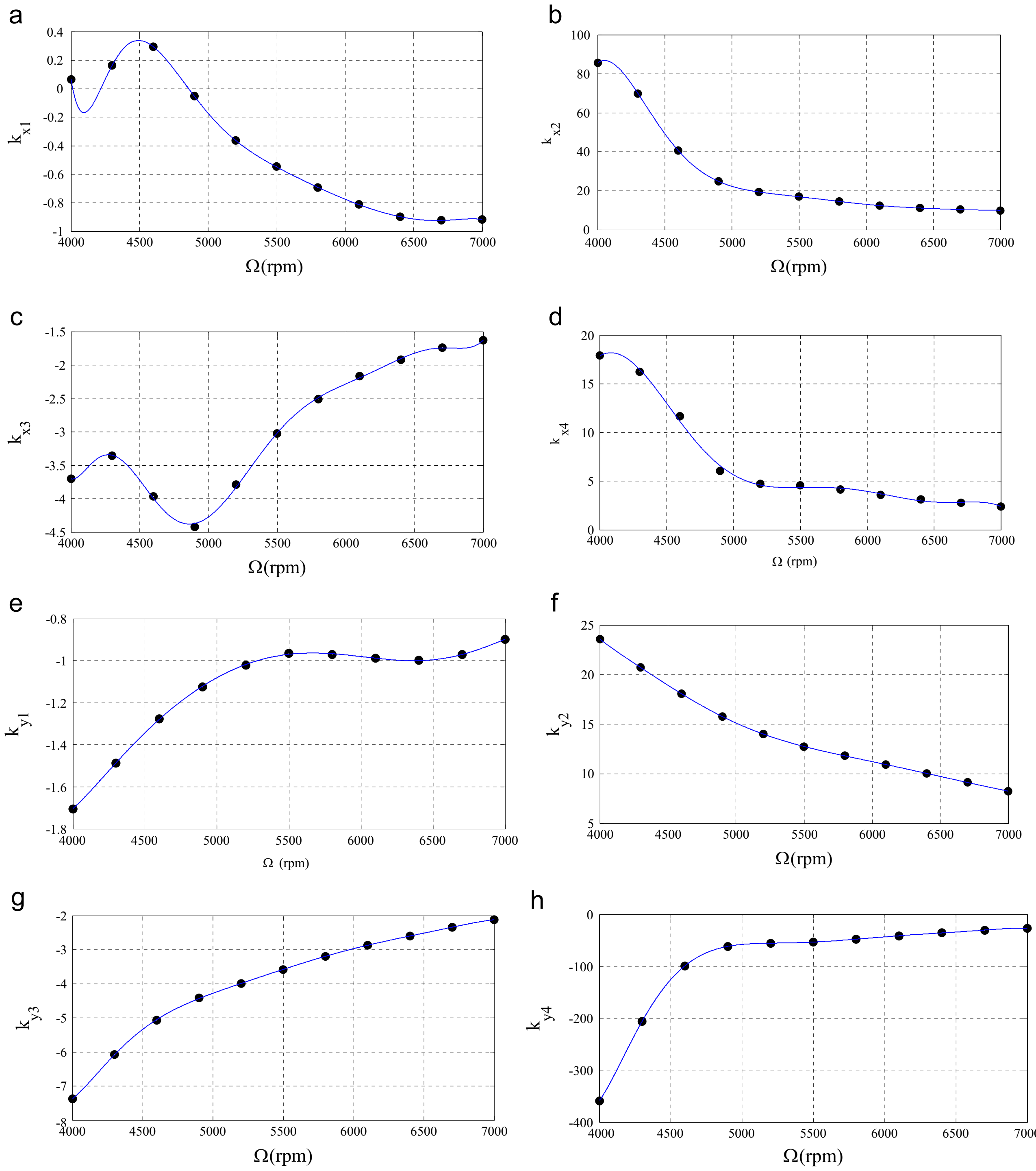

Fig. 3. Variations of dimensionless nonlinear dynamic coefficients with respect to journal angular velocity; identified coefficients: black "O", fitted polynomial: blue solid line, (a) k_{x1}; (b) k_{x2}; (c) k_{x3}; (d) k_{x4}; (e) k_{y1}; (f) k_{y2}; (g) k_{y3}; (h) k_{y4}.

The oil film thickness expression, Eq. (3), and its spatial derivative with respect to θ, are linearly dependent on the DOFs of the TPJB. Also, a linear relationship holds between the temporal derivative of the thickness expression and the velocities corresponding to the TPJB's DOFs. Thus, the first and the second terms on the right hand side of the Reynolds equation are linear functions of displacements and velocities of the TPJB's DOFs, respectively. On the other hand, since cubic terms of oil film thickness are included in the linear operator, the operator would be a third order polynomial function of the displacements of the DOFs of the TPJB. Therefore, the inverse of the operator described in Eq. (14) may be expanded as an infinite power series in displacements of TPJB's DOF, known as a Taylor series expansion. According to the above statements, it is clarified that the pressure distribution as evaluated by Eq. (14), may be expressed by a Taylor series expansion including terms which have two features: first, they are linear functions of velocities of TPJB's DOF; and second, they can be comprised of products of displacements of TPJB's DOF up to higher orders. It should be noticed that the pads motion are related to the journal motion in the proposed nonlinear model of the TPJB. In dynamic analysis of TPJBs, the tilting angles of pads are considered as independent degrees of freedom and the whole journal-pad dynamics have to be treated generally as a coupled

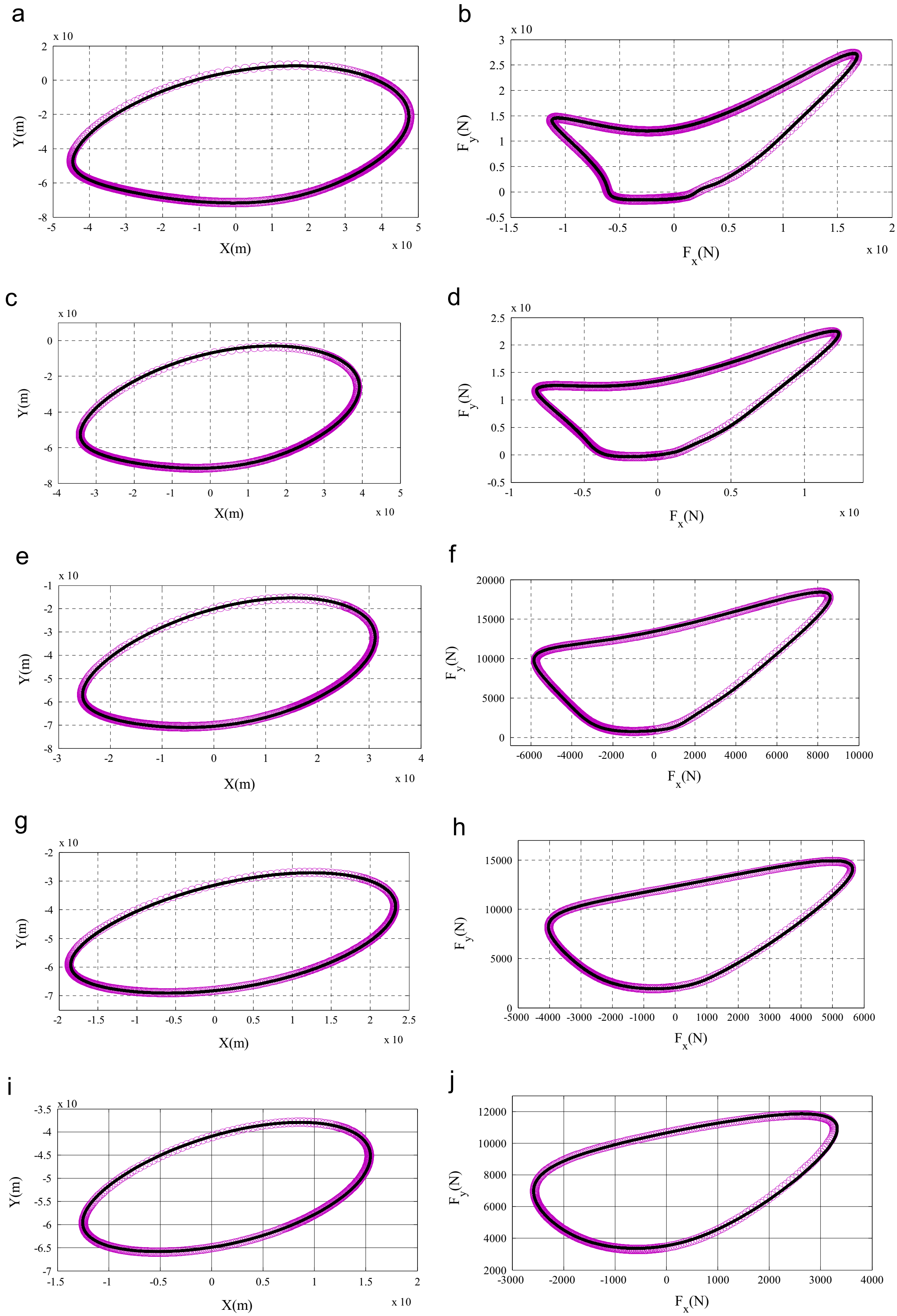

Fig. 4. Simulated journal orbit, (X,Y) and oil film forces (F_X,F_Y) at $\Omega=5950$ rpm; original model: magenta "O", nonlinear model: black solid line: (a and b) $e=3$ μm; (c and d) $e=2.5$ μm (e and f) $e=2$ μm; (g and h) $e=1.5$ μm (i and j) $e=1$ μm. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

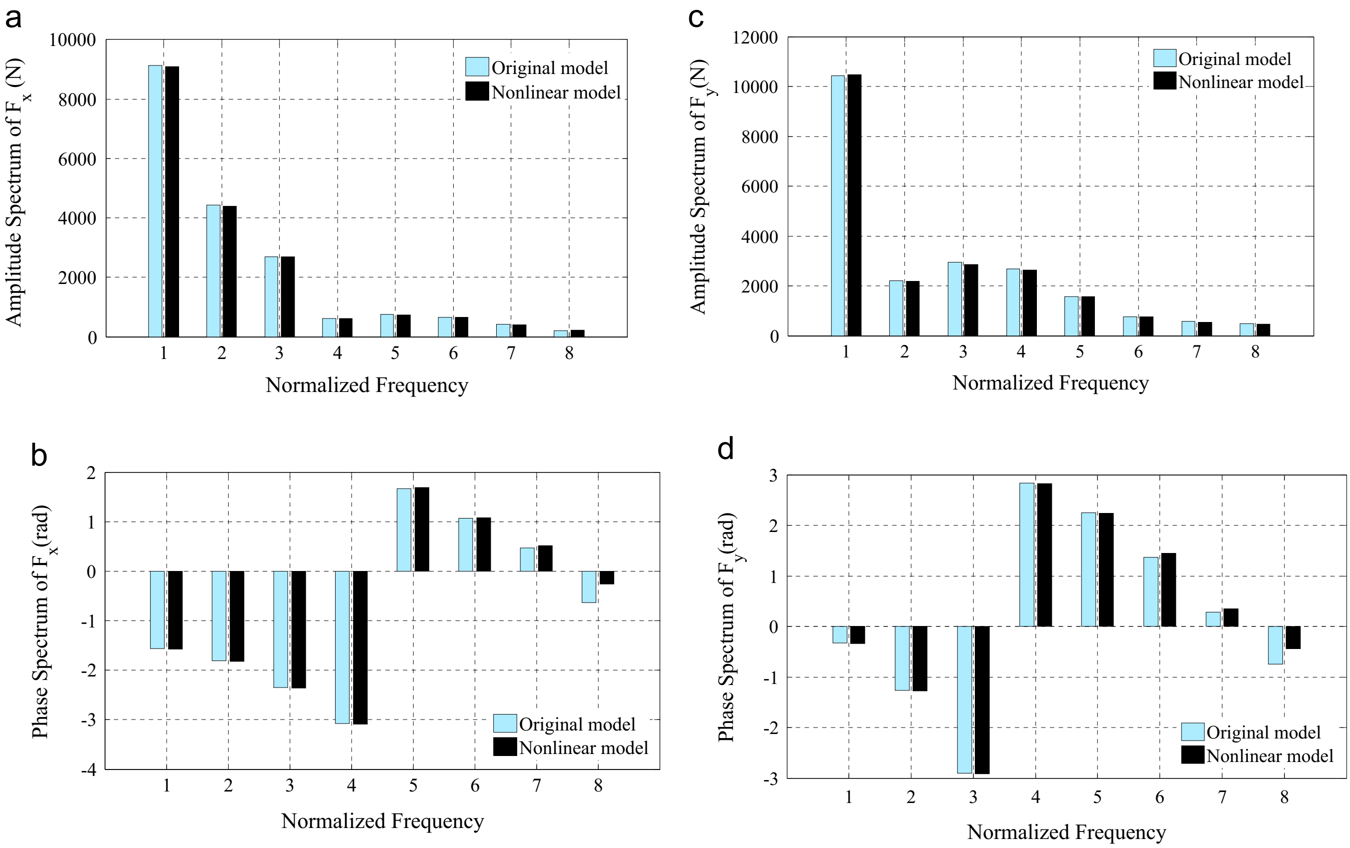

Fig. 5. Comparison between the spectrum analysis of the oil film forces obtained from the two models at the angular velocity, $\Omega = 5950$ rpm, and the unbalance excitation, $e = 3$ μm (original model: blue, nonlinear model: black): (a) amplitude of F_x; (b) phase of F_x; (c) amplitude of F_y; (b) phase of F_y. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

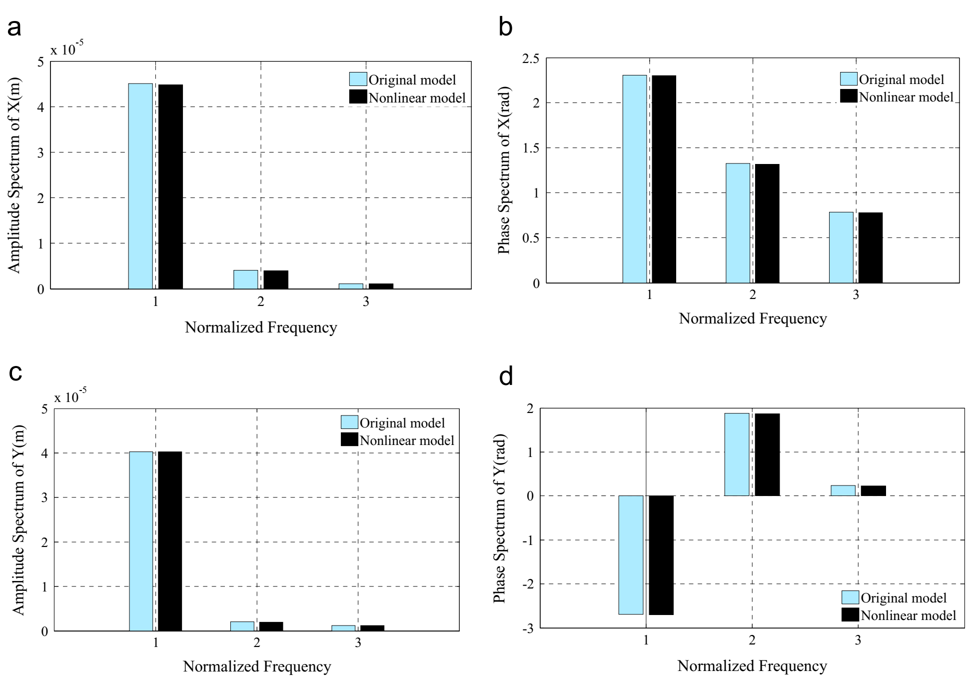

Fig. 6. Comparison between the spectrum analysis of the journal displacements obtained from the two models at the angular velocity, $\Omega = 5950$ rpm, and the unbalance excitation, $e = 3$ μm (original model: blue, nonlinear model: red): (a) amplitude of X; (b) phase of X; (c) amplitude of Y; (b) phase of Y. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

vibrating system. However, under specific conditions considering three assumptions, stated in Section 2, nonlinear dynamic terms of the oil film forces as well as the linear dynamic terms are described by polynomial series incorporating, only, journal DOFs displacements and velocities products. This order reduction is proposed as the main contribution of the present research. By using the reduced nonlinear

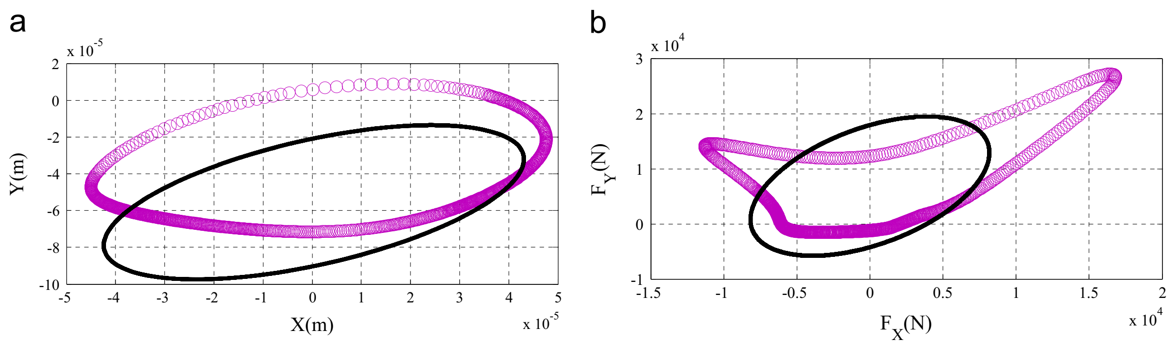

Fig. 7. Simulated data at $\Omega=5950$ rpm and $e=3$ μm, original model: magenta "O", linear model: black solid line: (a) journal orbit; (b) oil film forces. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

model, the time required for nonlinear analysis of TPJBs will be reduced significantly compared to the time needed for nonlinear analysis by the original model described in Section 2. Moreover, desired computational accuracy may be achieved by choosing an appropriate order for Taylor series used for representing the nonlinear parts of the oil film forces. Thus in the followings, only the DOFs associated with the journal translational motions will be included in the model which relates the pressure distribution to the journal's DOFs by a Taylor series expansion. The described Taylor series may be represented with the aid of following double product series of journal displacement DOFs:

$$(X\times Y)^n=\left\{X^n \quad X^{n-1}Y \quad \cdots \quad XY^{n-1} \quad Y^n\right\};n\geq 2$$
$$\left(\dot{X}\times X\times Y\right)^n=\left\{\dot{X}X^n \quad \dot{X}X^{n-1}Y \quad \cdots \quad \dot{X}XY^{n-1} \quad \dot{X}Y^n\right\};n\geq 1$$
$$\left(\dot{Y}\times X\times Y\right)^n=\left\{\dot{Y}X^n \quad \dot{Y}X^{n-1}Y \quad \cdots \quad \dot{Y}XY^{n-1} \quad \dot{Y}Y^n\right\};n\geq 1 \tag{15}$$

Taylor series expansions including journal's DOFs products up to order n, used for expressing the nonlinear parts of the oil film forces is described as,

$$F_x^{NL}=\left[(X\times Y)^2 \quad \cdots \quad (X\times Y)^n \quad \left(\dot{X}\times X\times Y\right)^1 \right.$$
$$\left.\cdots\left(\dot{X}\times X\times Y\right)^{n-1}\left(\dot{Y}\times X\times Y\right)^1\cdots\left(\dot{Y}\times X\times Y\right)^{n-1}\right]\{K\}^X$$
$$F_y^{NL}=\left[(X\times Y)^2 \quad \cdots \quad (X\times Y)^n \quad \left(\dot{X}\times X\times Y\right)^1 \right.$$
$$\left.\cdots\left(\dot{X}\times X\times Y\right)^{n-1}\left(\dot{Y}\times X\times Y\right)^1\cdots\left(\dot{Y}\times X\times Y\right)^{n-1}\right]\{K\}^Y \tag{16}$$

Therefore, one may rewrite the equation of motion of a rigid rotor supported by a TPJB defined in Eq. (1) in a reduced form:

$$\begin{cases} m_J\ddot{X}=F_x+m_Je\Omega^2\cos\Omega t \\ m_J\ddot{Y}=F_y+m_Je\Omega^2\sin\Omega t-m_Jg \end{cases} \tag{17}$$

In the following section, the procedure for identification of the reduced nonlinear model will be presented. The numerically simulated transient responses of a rigid rotor supported on TPJB are used to identify the speed dependent nonlinear dynamic coefficients. Then, the unbalance response evaluated through using the identified reduced model will be verified by comparison with the one obtained through applying the original model of the TPJB.

4. Results and discussion

The specifications of the TPJB under study utilized in supporting parts of an industrial gas turbine are introduced in Table 1. THD analysis of the TPJB has been performed to determine equilibrium positions of the TPJB, corresponding temperature distributions and synchronized linear dynamic coefficients in a journal angular velocity range, from 4000 to 7000 rpm with 300 rpm increments.

The transient analysis of the TPJB, through numerical integration of Eq. (1), representing the original model, have been implemented in the journal angular velocity range, and, in five levels of unbalance excitation, 1 to 3 μm at 0.5 μm intervals. The differential solver routine ode45 is used for numerically integrating the TPJB original model equations. The displacement and velocity initial conditions for each transient analysis are set to the related equilibrium position and zero, respectively. Oil film viscosity variation for each transient analysis is considered as the one obtained at related equilibrium positions of the TPJB. The simulated data, including time series of displacements, velocities and oil film forces, are employed to set up an over determined linear systems of equations, using Eqs. (11) and (16).

One may represent these two systems of linear equations, corresponding to nonlinear parts of the oil film forces in x and y directions, as:

$$\begin{cases} [X]\{K\}_x=\{f\}_x \\ [X]\{K\}_y=\{f\}_y \end{cases} \tag{18}$$

It should be noticed that for determining the right hand side of Eq. (18), linear parts of the oil film forces have been obtained through evaluating the linear synchronous dynamic coefficients. The systems of Eq. (18) are solved to estimate the nonlinear dynamic model. The dominant terms in the nonlinear model are distinguished using the subset selection technique introduced in [32]. Application of this technique to the parameter estimation problem, sorts the terms according to their contributions in reproducing the simulated oil film forces in a descending order.

The variations of residues with respect to ordered terms obtained from application of subset selection to the parameter estimation problem, Eq. (18), at an angular velocity, $\Omega=5500$ rpm, and Taylor series to the order of thirteen, $n=13$, is shown in Fig. 2. The residue is defined as the norm2 of difference between the oil film force vectors obtained through using the exact value and the ones evaluated through using the reduced model, divided by the norm2 of oil film force vectors obtained through using the exact value.

Choosing the first forty terms obtained through applying the subset selection to the parameter estimating problem, in order to

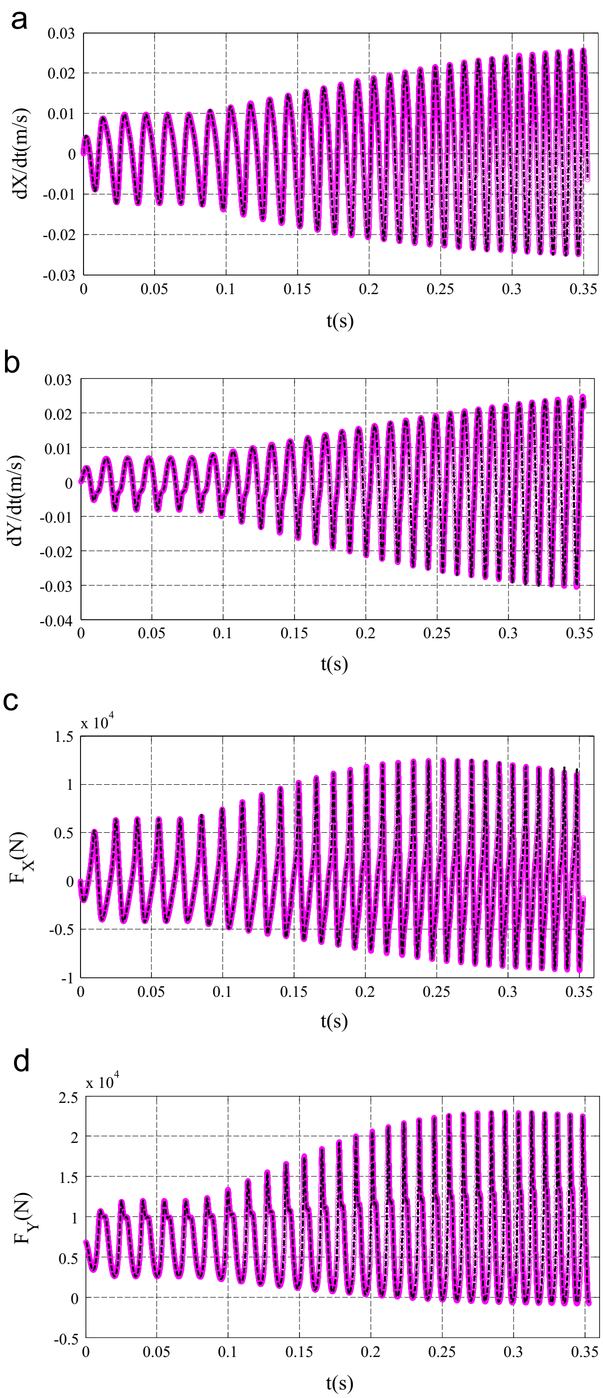

Fig. 8. Transient responses exposed to unbalance excitations, $e=2.5$ μm, during run-up with constant angular acceleration over the identification range (original model: magenta ".", nonlinear model: black dash line): (a) velocity in x direction vs. time; (b) velocity in y direction vs. time; (c) oil film force in x direction vs. time; (d) oil film force in y direction vs. time. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

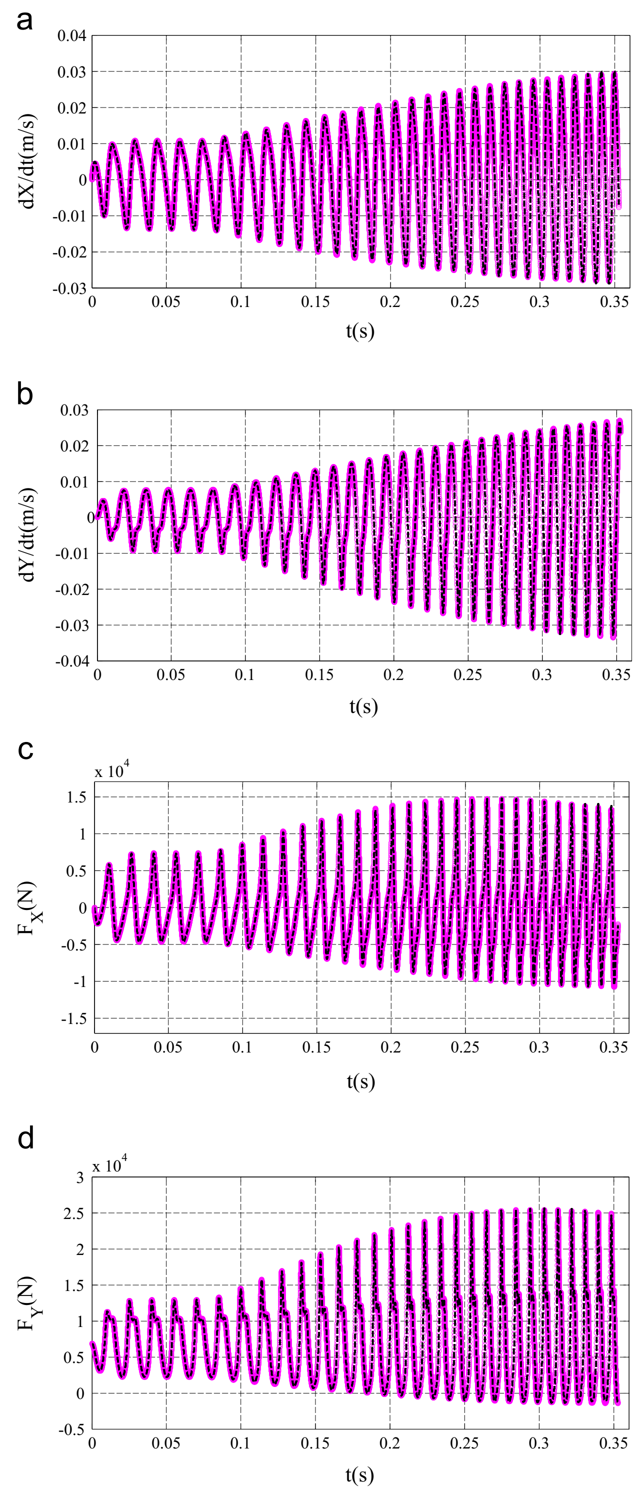

Fig. 9. Transient responses exposed to unbalance excitations, $e=2.75$ μm, during run-up with constant angular acceleration over the identification range (original model: magenta ".", nonlinear model: black dash line): (a) velocity in x direction vs. time; (b) velocity in y direction vs. time; (c) oil film force in x direction vs. time; (d) oil film force in y direction vs. time. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

keep the relative residues less than 2%, the nonlinear model is represented as:

$$F_x^{NL} = \begin{bmatrix} X^2 & \dot{Y}X^4 & \dot{X}Y & \dot{X}Y^2 & \cdots \end{bmatrix} \begin{Bmatrix} K_{x1} \\ K_{x2} \\ K_{x3} \\ K_{x4} \\ \vdots \end{Bmatrix}^X$$

$$F_y^{NL} = \begin{bmatrix} X^2 & \dot{X}XY & \dot{Y}Y & XY^4 & \cdots \end{bmatrix} \begin{Bmatrix} K_{y1} \\ K_{y2} \\ K_{y3} \\ K_{y4} \\ \vdots \end{Bmatrix}^Y \quad (19)$$

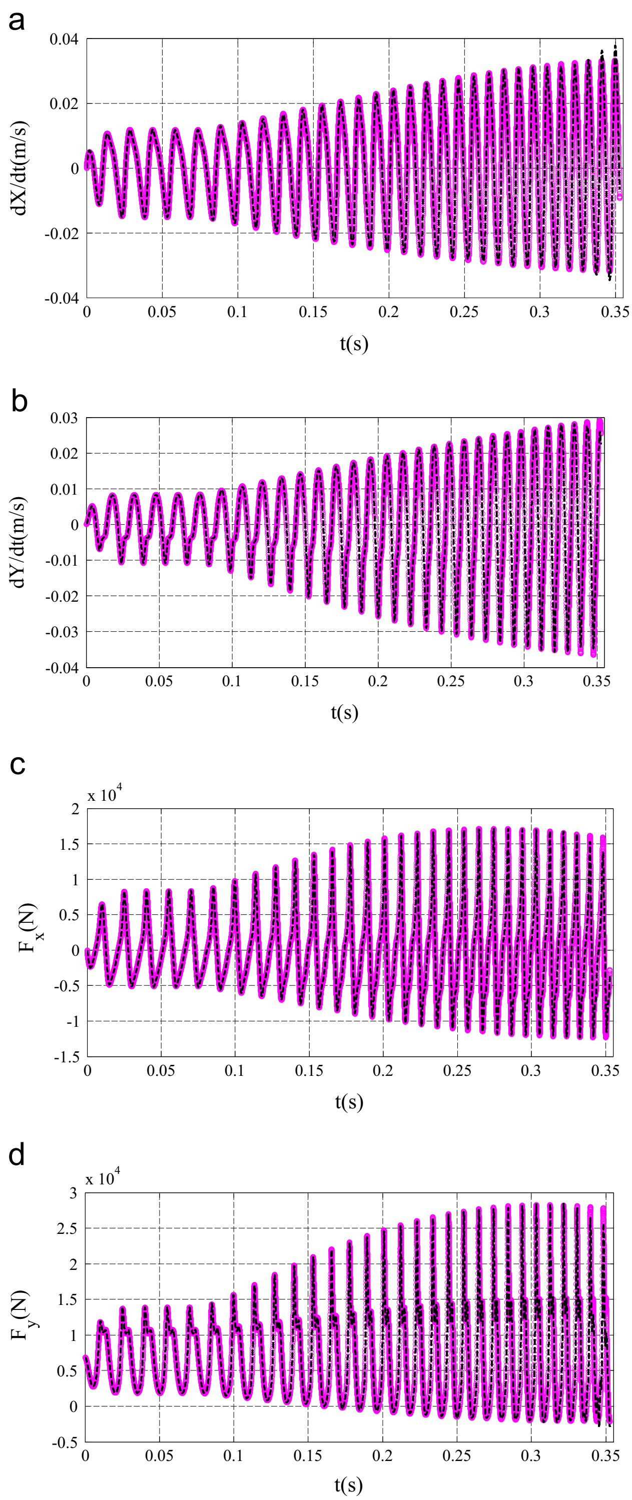

Fig. 10. Transient responses exposed to unbalance excitations, $e=3\ \mu\text{m}$, during run-up with constant angular acceleration over the identification range (original model: magenta ".", nonlinear model: black dash line): (a) velocity in x direction vs. time; (b) velocity in y direction vs. time; (c) oil film force in x direction vs. time; (d) oil film force in y direction vs. time. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The variations of the dimensionless nonlinear dynamic coefficients, mentioned in Eq. (19), with respect to the journal angular velocity is seen in Fig. 3.

One notices in Fig. 3 the identified coefficients changes continuously on the depicted angular velocity interval. Thus, the nonlinear coefficients at an angular velocity not included in the identification range, may be well approximated by polynomial curve fitting to the identified ones depicted in Fig. 3. The identified nonlinear model have been used to predict the steady state journal orbit and nonlinear oil film forces at an angular velocity, $\Omega=5950$ rpm which is not included in the specified range. The journal orbit and oil film forces obtained from numerical integration of the original model, described by Eq. (1), are compared in Fig. 4, with the ones evaluated by applying the identified nonlinear model to the equations of motion, Eq. (17).

The comparison of spectrum analysis of the unbalance response obtained at an angular velocity, $\Omega=5950$ rpm, and unbalance excitation, $e=3\ \mu\text{m}$, by employing the original model with the one evaluated through using the reduced nonlinear model of the TPJB are displayed in Figs. 5 and 6. It is noticed, in Figs. 5 and 6, the evaluated amplitude and phase of the oil film forces and journal displacements corresponding to each harmonic term are in good agreements with their counterparts. Thus, the responses obtained by using the original model and the nonlinear model have the same frequency content. Excellent match between two sets of results, shown in Figs. 4–6, indicates that the nonlinear model is an appropriate alternative for the original model at a constant angular velocity within the range in which the identification is performed.

Using the new model which expands the bearing forces by the journal harmonic motion, one may conveniently employ harmonic balance method to evaluate rotor unbalance responses dominated by weak nonlinear effects. Moreover, the proposed model may be well incorporated into governing equations of a flexible rotor supported by TPJBs to predict shifts in related critical speeds and unbalance responses.

It is noteworthy that, as depicted in Fig. 7, the reduced model incorporating, only, the linear dynamic coefficients represent very poorly rotating system dynamics obtained through using the original model.

Unbalance response evaluation of the rigid rotor exposed to unbalance excitations, $e=2.5$, 2.75, 3 μm during run-up over the evaluated range of angular velocity with constant angular acceleration, $A=648{,}000\ \text{rev/min}^2$, have been implemented. The journal velocities and oil film forces in x and y directions obtained from numerical simulation incorporating the original model are compared with the ones obtained through using the identified nonlinear model, in Figs. 8–10. Good agreement between the results is achieved and only slight differences are seen between transient responses in last three cycles corresponding to highest load condition, i.e. $e=3\ \mu\text{m}$. Thus, transient response of a rotor supported by a TPJB with constant angular acceleration can be appropriately simulated using the reduced nonlinear model.

5. Conclusion

The present study proposes a new methodology for nonlinear modeling and identification of TPJBs in order to construct an accurate reduced order model with weak nonlinear effects. The nonlinear model is based on a Taylor series expansion of oil film forces with respect to displacements and velocities of the journal center. The nonlinear terms incorporated in the Taylor series expansion has been developed in such a way to be consistent with the transient form of Reynolds differential equation. A numerical integration of the governing equations of a rigid rotor supported by a TPJB, in a range of angular velocities with and without constant angular acceleration and in different unbalance loads, has been implemented to provide sufficient numerical data to be utilized in nonlinear model parameter estimation. The subset selection technique has been employed to organize the terms in the nonlinear model in a descending order, which characterizes their contributions in estimating the simulated oil film forces. The identified nonlinear model is validated by comparing its predicated responses with the ones evaluated by applying the original model to the equations of motion. Good correlations between the two sets of results have been achieved in both time and frequency domains. The proposed nonlinear model provides a fast computational tool for analysts to investigate the nonlinear behavior in rotating systems supported on TPJBs.

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