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DAMAGE LOCATION INDICATORS FROM SUBSTRUCTURE MODE SHAPES

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When damage occurs in a substructure its modes of vibration will be changed but the modes of other substructures will be unaffected. Also the higher modes will not participate in the deflections of undamaged substructures. These observations are used to establish the necessary and sufficient conditions that identify an undamaged substructure. Two damage location indicators are then formulated and applied to a numerical example. Both indicators are successful in locating the damage.

Keywords: Structural damage; vibration; nondestructive eualuation

1. INTRODUCTION

Modern approaches to condition monitoring need reliable procedures for the location and assessment of damage by using information gathered from machines and structures. This paper will concentrate on the use of low frequency vibration measurements for damage detection, which is an area that has received considerable attention in recent times (Natke and Cempel [1], Doebling *et al.* [2]). Furthermore the discussion will be limited to model-based approaches which rely on the deviation of the measurements from predictions, typically

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obtained from a finite element model, to assess the location and extent of damage in a structure.

Pandey, Biswas and Samman [3] compared the curvature of the mode-shape of a beam in its damaged condition with the mode-shape curvature of an undamaged beam. Zimmerman and Kaouk [4] considered change in stiffness to be responsible for the deviation of the 'damaged' eigenvalue equation from its finite element ('undamaged') counterpart. They used the angle between a damaged eigenvector and a row of the matrix $(\lambda_d^2\mathbf{M} + \lambda_d\mathbf{C} + \mathbf{K})$ to locate the damage. The matrices **M**, **C** and **K** have the usual meanings and λ_d denotes an eigenvalue of the damaged system. Stubbs and Kim [5] described a method based on the observation that modal energies remained largely unchanged at locations remote from the damage. They developed a damage location indicator and a damage severity indicator by using the modal stiffness contributions from sub-members of the structure. Rytter [6] classified the damage detection problem in four levels, namely: (i) identify that damage has occurred; (ii) identify that damage has occurred and determine the location of the damage; (iii) identify that damage has occurred, locate the damage and estimate its severity; (iv) identify that damage has occurred, locate the damage, estimate its severity and determine the remaining useful life of the structure.

In this paper we consider only the Level (ii) damage location problem. The task of determining the nature and extent can be easily and more efficiently done later by using X-ray, optical or ultrasonic methods. The method described here proceeds by firstly dividing the structure into several substructures. Then certain criteria are applied to establish which of the substructures are damage free, and by elimination the location of the damaged substructures.

One way of establishing if a substructure is undamaged is to show that its mode shapes have not been changed. Also, when there is no damage to the substructure in question, but elsewhere there may be damage, then the mode shapes of the highest eigenvalues of the substructure will not be expected to participate in the measured substructure displacements. In what follows we parameterize the changes in the substructure mode shapes (rather than parameterizing the characteristics of the damage) and establish two indicators for damage location. It is demonstrated that both indicators will tend to vanish when the substructure in question is undamaged. A numerical example is used to illustrate the performance of the two indicators.

Although the method development concentrates on substructures and substructure modes, it should be emphasized that this method takes the global mode shapes of the structure and locates damage by considering each substructure in turn to determine whether this substructure is damaged. Thus the approach differs from a direct method that uses all the mode shape data simultaneously, however all the mode shape data is used as the technique focuses on individual substructures sequentially. The method is demonstrated on a frame structure but is general and may be applied to any structure.

2. THEORY

We consider a structure in vibration with global displacements $\mathbf{u}_g$. The displacement of part of the structure (the substructure) may be assembled in the vector $\mathbf{u} \in \Re^N$. Predictions from an existing analytical (finite element) model, for the substructure vector, are also available and denoted by $\mathbf{u}_0$.

The substructure displacement vector can be related to both the substructure mode shapes of the physical system and those predicted by the model. Thus,

$$\mathbf{u} = \mathbf{\Phi}\mathbf{p}, \quad \mathbf{u}_0 = \mathbf{\Phi}_0\mathbf{p}_0 \tag{1,2}$$

where $\mathbf{\Phi}$ is the matrix of normal (undamped) mode shapes of the substructure under free end conditions, $\mathbf{p}$ is the vector of participation factors, and the subscript 0 denotes values predicted by the analytical model. The method does not require the physical measurement of the substructure mode shapes, but we observe that they are given by a linear combination of their finite element counterparts,

$$\mathbf{\Phi} = \mathbf{\Phi}_0\mathbf{R}, \tag{3}$$

where $\mathbf{R}$ is a rotation matrix. The difference between the observed displacement vector and the finite element prediction is given, from Eqs. (1)–(3), as,

$$\Delta\mathbf{u} = \mathbf{u} - \mathbf{u}_0 = \mathbf{\Phi}_0(\mathbf{R}\mathbf{p} - \mathbf{p}_0), \tag{4}$$

where the right-hand-side contains the participation factors in terms of the analytical modes.

The vectors $\mathbf{p}$ and $\mathbf{p}_0$ will be different whenever there is damage in any part of the structure and not just when the damage is confined to the local substructure. On the other hand, $\mathbf{R}$ will only deviate from the identity matrix when the local substructure is damaged. This observation is the basis of the method set forth in this article.

$\mathbf{R}$ and $\mathbf{p}$ may be expressed as,

$$\mathbf{R} = \mathbf{I} + \Delta\mathbf{R}, \quad \mathbf{p} = \mathbf{p}_0 + \Delta\mathbf{p} \tag{5,6}$$

where $\Delta\mathbf{R}$ and $\Delta\mathbf{p}$ represent the changes in the mode shapes and participation factors respectively. By combining Eqs. (4)–(6) it is found that,

$$\Delta\mathbf{u} = \mathbf{\Phi}_0(\Delta\mathbf{p} + \Delta\mathbf{R}\,\mathbf{p}_0 + \Delta\mathbf{R}\,\Delta\mathbf{p}) \tag{7}$$

We now parameterize the change in the substructure mode shapes so that,

$$\Delta\mathbf{R}\,\mathbf{p}_0 = \mathbf{A}\mathbf{x} \tag{8}$$

where $\mathbf{x} \in \Re^M$ defines the change $\Delta\mathbf{R}$, and M is the number of parameters required to parameterize $\Delta\mathbf{R}$. Equation (8) is merely a re-arrangement of terms, where unknown elements in $\Delta\mathbf{R}$ transfer to $\mathbf{x}$ and elements of $\mathbf{p}_0$ transfer to $\mathbf{A} \in \Re^{N \times M}$. This is an important step in the method because we aim to detect a change in the mode-shapes of a substructure independently of the particular damage characteristics. We only need to determine whether $\mathbf{x}$ contains any non-zero terms whereupon it is assured that the local substructure is damaged, and it is unnecessary to parameterize the physical characteristics of the damage. The latter is often required in model-based damage detection procedures and is a major drawback of those methods because it presupposes the nature of the damage (*e.g.*, a crack with dimensions that can be parameterized).

Equation (7) can be re-written by using the relationship (8) to give,

$$\mathbf{\Phi}_0(\Delta\mathbf{p} + \mathbf{A}\mathbf{x}) - \Delta\mathbf{u} = -\mathbf{\Phi}_0\,\Delta\mathbf{R}\,\Delta\mathbf{p} \tag{9}$$

and can be arranged in matrix form as,

$$[\mathbf{\Phi}_0 \vdots \mathbf{\Phi}_0\mathbf{A} \vdots \mathbf{\Delta u}] \begin{Bmatrix} \mathbf{\Delta p} \\ \mathbf{x} \\ -1 \end{Bmatrix} = -\mathbf{\Phi}_0\, \mathbf{\Delta R}\, \mathbf{\Delta p}. \tag{10}$$

It is useful to emphasize the known and unknown terms in Eq. (10). $\mathbf{\Phi}_0$ is known from the analytical model, $\mathbf{A}$ is known from the participation factors of the analytical modes *via* Eq. (8) and $\mathbf{\Delta u}$ is known from the measurements and the analytical model. The unknowns are $\mathbf{\Delta p}$ and $\mathbf{x}$ (which may be used to generate $\mathbf{\Delta R}$).

One effect of damage in a substructure is to produce sharp changes in the slopes of the substructure displacements $\mathbf{u}$. This means that the higher modes, $\tilde{\mathbf{\Phi}}_0$, of the local substructure will participate in $\mathbf{\Delta u}$ so that their contribution can be expressed as,

$$\mathbf{\Delta\tilde{u}} = \tilde{\mathbf{\Phi}}_0\, \mathbf{\Delta\tilde{p}}. \tag{11}$$

But in general the higher modes are not sensitive to damage in other substructures. Based on this observation we assert that for an undamaged substructure,

$$\tilde{\mathbf{\Phi}}_0^T\, \mathbf{M}_0\, \mathbf{\Delta u} = 0, \tag{12}$$

where the modes $\mathbf{\Phi}_0$ are normalized by the substructure mass matrix $\mathbf{M}_0$.

We observe from Eq. (10) that when $(\mathbf{\Delta p}^T, \mathbf{x}^T, -1)^T$ spans the null-space of $[\mathbf{\Phi}_0 \vdots \mathbf{\Phi}_0\mathbf{A} \vdots \mathbf{\Delta u}] \in \Re^{N\times(N+M+1)}$ then the right hand side will be zero. Also, *null* $[\mathbf{\Phi}_0 \vdots \mathbf{\Phi}_0\mathbf{A} \vdots \mathbf{\Delta u}]$ is the subspace of solutions that contains, $\mathbf{x}=\mathbf{0}$, $\mathbf{\Delta\tilde{p}} = \mathbf{0}$ because, on the right hand side of Eq. (10), $\mathbf{\Delta R}$ contains only the same terms that are rearranged in $\mathbf{x}$ so that when $\mathbf{x}=\mathbf{0}$ then $\mathbf{\Delta R}=\mathbf{0}$ identically. Therefore $\mathbf{\Phi}_0 \mathbf{\Delta R}\, \mathbf{\Delta p}=\mathbf{0}$ is a necessary condition for an undamaged substructure.

When the right-hand-side of Eq. (10) is zero a family of solutions for $\mathbf{x}$ and $\mathbf{\Delta p}$ (including $\mathbf{x}=\mathbf{0}$, $\mathbf{\Delta\tilde{p}} = \mathbf{0}$) can be written as

$$\mathbf{Va} = \begin{Bmatrix} \mathbf{\Delta p} \\ \mathbf{x} \\ -1 \end{Bmatrix}, \tag{13}$$

where the columns of $\mathbf{V} \in \Re^{(N+M+1)\times(M+1)}$ are the right singular vectors corresponding to the zero singular values of $[\boldsymbol{\Phi}_0 \vdots \boldsymbol{\Phi}_0\mathbf{A} \vdots \Delta\mathbf{u}]$, and the non-zero vector $\mathbf{a} \in \Re^{M+1}$ determines an arbitrary combination of the singular vectors.

We consider a subset of the Eq. (13), namely those related to $\mathbf{x}$ and the changes in the participation factors of the substructure higher modes $\Delta\tilde{\mathbf{p}}$,

$$\mathbf{V}_1\mathbf{a} = \begin{Bmatrix} \Delta\tilde{\mathbf{p}} \\ \mathbf{x} \end{Bmatrix}. \tag{14}$$

For an undamaged substructure both $\mathbf{x}$ and $\Delta\tilde{\mathbf{p}}$ (the entire right-hand-side of Eq. (14)) should be zero, so that it is sufficient ($\mathbf{a} \neq \mathbf{0}$) for $\mathbf{V}_1$ to be singular. Since $\mathbf{V}_1$ is a submatrix formed from the rows of $\mathbf{V}$, and $\mathbf{V}$ was established on the necessary condition that $\boldsymbol{\Phi}\Delta\mathbf{R}\Delta\mathbf{p} = \mathbf{0}$, it is apparent that the substructure will be undamaged if and only if,

$$\frac{1}{\mathrm{cond}(\mathbf{V}_1)} \to 0 \tag{15}$$

Thus the condition number of $\mathbf{V}_1$ can be used as an indicator,

$$i_1 = \frac{1}{\mathrm{cond}(\mathbf{V}_1)} \tag{16}$$

to locate a faulty substructure. In the analysis above it was assumed that a full length displacement vector of the substructure was available. In practice however it is likely that the measured displacement vectors will be incomplete and the measured mode shapes would have to be expanded or the model reduced. This is outside the scope of the present paper and will not be considered further.

Thus far only a single measurement or mode shape vector, $\Delta\mathbf{u}$, has been considered. One approach is to obtain the indicator i_1 for all the measured modes and candidate substructures, and compare the results. The alternative is to extend Eq. (10) as

$$\begin{bmatrix} \boldsymbol{\Phi}_0 & \mathbf{0} & \cdots & \vdots & \boldsymbol{\Phi}_0\mathbf{A}_1 & \vdots & \Delta\mathbf{u}_1 \\ \mathbf{0} & \boldsymbol{\Phi}_0 & \cdots & \vdots & \boldsymbol{\Phi}_0\mathbf{A}_2 & \vdots & \Delta\mathbf{u}_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{p}_1 \\ \Delta\mathbf{p}_2 \\ \vdots \\ \mathbf{x} \\ -1 \end{Bmatrix} = -\boldsymbol{\Phi}_0\Delta\mathbf{R} \begin{Bmatrix} \Delta\mathbf{p}_1 \\ \Delta\mathbf{p}_2 \\ \vdots \end{Bmatrix}. \tag{17}$$

where $\Delta\mathbf{u}_i$ denotes the ith measured displacement, which has a corresponding change in participation factor of $\Delta\mathbf{p}_i$. Notice that the matrix $\Delta\mathbf{R}$ (and therefore vector $\mathbf{x}$), which determines the change in substructure mode shapes, is assumed to be the same for all measured vectors. The method, and the indicator i_1, then follow naturally by considering the null space of

$$\begin{bmatrix} \mathbf{\Phi}_0 & \mathbf{0} & \cdots & \vdots & \mathbf{\Phi}_0\mathbf{A}_1 & \vdots & \Delta\mathbf{u}_1 \\ \mathbf{0} & \mathbf{\Phi}_0 & \cdots & \vdots & \mathbf{\Phi}_0\mathbf{A}_2 & \vdots & \Delta\mathbf{u}_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

It should be noted that a further indicator,

$$i_2 = \frac{\tilde{\varphi}_0^T \mathbf{M}_0 \, \Delta\mathbf{u}}{\sqrt{\Delta\mathbf{u}^T \mathbf{M}_0 \, \Delta\mathbf{u}}}, \tag{18}$$

may be obtained from Eq. (12). The vector $\tilde{\varphi}_0$ is given by any linear combination of the higher modes given in the columns of $\tilde{\mathbf{\Phi}}_0 = [\tilde{\phi}_1, \tilde{\phi}_2, \ldots, \tilde{\phi}_n]$. Thus,

$$\tilde{\varphi}_0 = \alpha_1 \tilde{\phi}_1 + \alpha_2 \tilde{\phi}_2 + \cdots + \alpha_n \tilde{\phi}_n, \tag{19}$$

where,

$$\alpha_1^2 + \alpha_2^2 + \cdots + \alpha_n^2 = 1 \tag{20}$$

The indicator i_2 appears to be less stringent than i_1 because it is based only on the participation of the higher modes and not on the invariance of the substructure modes to damage elsewhere. It should also be emphasized that the measures are relative in the sense that the most likely damage site will be identified. The threshold to determine whether damage has occurred or not has to be chosen on the basis of experience. Ideally a threshold could be specified analytically, but unfortunately this is not possible for the measures suggested. With the correct eigenvector scaling both indicators are constrained to be between zero and one, but generally the indicators will be much less than one. In the following section we investigate the performance of the two indicators using a numerical example.

3. SIMULATED EXAMPLE

The example is the cantilevered 3-bay truss shown in Figure 1, which was the subject of a GARTEUR benchmark recently [7, 8]. Each spar is modelled using a two Euler–Bernoulli beam-type finite elements.

The elements have the following properties: flexural rigidity, $EI = 5.67 \times 10^9\,\mathrm{Nm}^2$, mass density, $\rho = 2800\,\mathrm{kg/m}^3$ for all elements; cross sectional area, $A = 0.4 \times 10^{-2}\,\mathrm{m}^2$, length $\ell = 5\,\mathrm{m}$ for the horizontal spars; $A = 0.6 \times 10^{-2}\,\mathrm{m}^2$, $\ell = 3\,\mathrm{m}$ for the vertical spars; and $A = 0.3 \times 10^{-2}\,\mathrm{m}^2$ for the diagonal spars. The in-plane vibrations of the truss are considered, and the first eight modes of the undamaged structure (with natural frequencies in Hz) are shown in Figure 2. Two cases of damage are simulated; Case 1 is a 5% reduction in the stiffness of element 5, and Case 2 is a 5% reduction in the stiffness of both elements 5 and 10. The first five modes will be used for damage location.

The structure is divided into 12 substructures representing each spar of the truss. Table I shows the allocation of elements to substructures. Each substructure has 3 nodes and 9 degrees of freedom, so that for free end conditions there will be 3 rigid body modes and 6 strain modes. The rotation matrix $\mathbf{R}$ is an orthogonal matrix when both $\mathbf{\Phi}$ and $\mathbf{\Phi}_0$ are normalized with respect to $\mathbf{M}_0$, as is the case when the damage affects only the stiffness matrix. In our particular case $\mathbf{R}$ will be expressed in the form,

$$\mathbf{R} = \left[\begin{array}{ccc|cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0 & 0 & 0 & -x_1 & 1 & x_6 & x_7 & x_8 & x_9 \\ 0 & 0 & 0 & -x_2 & -x_6 & 1 & x_{10} & x_{11} & x_{12} \\ 0 & 0 & 0 & -x_3 & -x_7 & -x_{10} & 1 & x_{13} & x_{14} \\ 0 & 0 & 0 & -x_4 & -x_8 & -x_{11} & -x_{13} & 1 & x_{15} \\ 0 & 0 & 0 & -x_5 & -x_9 & -x_{12} & -x_{14} & -x_{15} & 1 \end{array}\right].$$

Using this transformation, the rigid body modes are unchanged. These 15 parameters x_i should all be zero if the substructure in question is undamaged. The matrix is orthogonal to first order in the parameters, but if large changes in the modes are expected then a

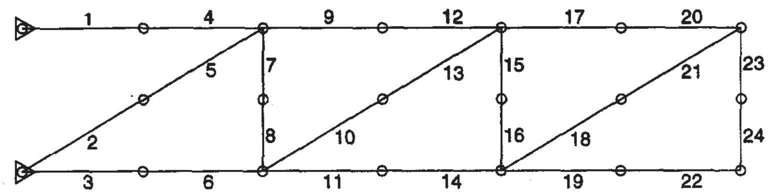

FIGURE 1 The example 3 bay truss. The element numbers are indicated. Circles represent the nodes and the triangles the fixed nodes.

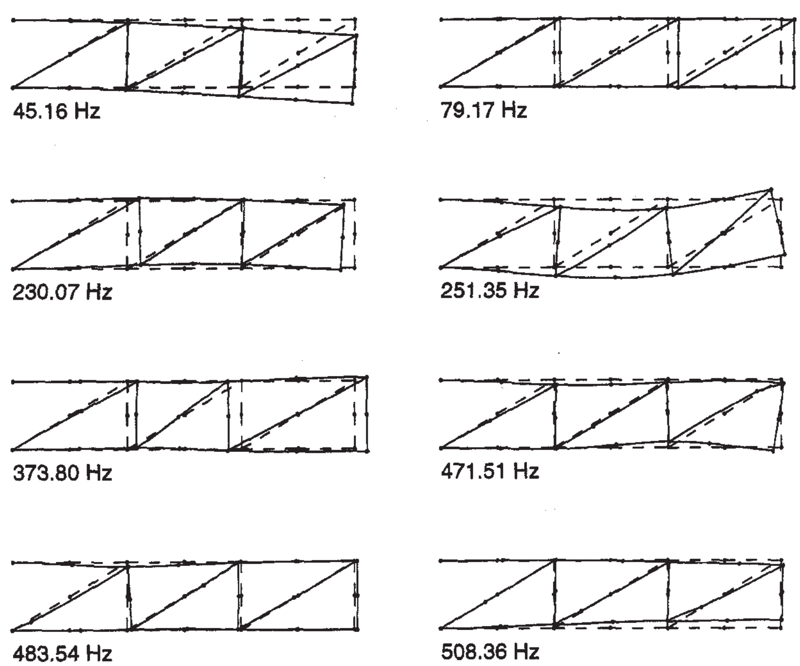

FIGURE 2 The first 8 natural frequencies and modes of the undamaged truss.

TABLE I The allocation of the elements to substructures

Substructure number	*Elements*	*Substructure number*	*Elements*
1	1, 4	7	11, 14
2	2, 5	8	15, 16
3	3, 6	9	17, 20
4	7, 8	10	18, 21
5	9, 12	11	19, 22
6	10, 13	12	23, 24

more general form of **R** would have to be used. Often the number of parameters may be reduced by assuming a particular form of **R**, for example by assuming that the symmetric and asymmetric modes of the substructure do not couple.

The choice of high frequency modes to use has already been considered. The modes should be the higher frequency modes of the substructure which do not participate in the global undamaged modes of the structure very much. For the GARTEUR frame example modes 8 and 9 satisfy this requirement for all of the substructures, and these modes will be considered as candidate modes for $\tilde{\mathbf{\Phi}}_0$.

Case 1 Figures 3 to 5 show the result of using the i_1 indicator. Note that substructure 2 has been damaged. In Figure 3 each global mode is treated independently, and mode 8 of the substructure is used for $\tilde{\mathbf{\Phi}}_0$. Clearly the damage is correctly located in substructure 2. Figure 4 shows the effect of using modes 8 and 9 of the substructure for $\tilde{\mathbf{\Phi}}_0$. The damage is located, but not as clearly as in Figure 3. It seems that mode 8 is a better mode to use for location than mode 9. Figure 5 shows the effect of using all five global measured modes simultaneously (using Eq. (17)), and using modes 8 and 9 of the substructure for $\tilde{\mathbf{\Phi}}_0$. Once again the damage is clearly located.

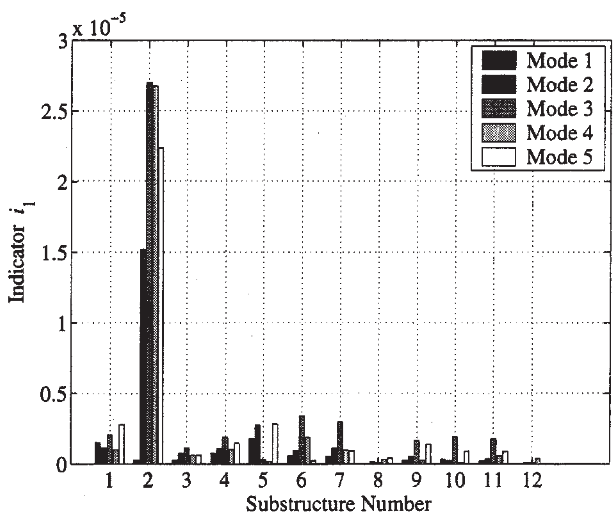

FIGURE 3 The indicator i_1 for Case 1. The indicator is calculated for the first five modes individually. $\tilde{\mathbf{\Phi}}_0 = [\phi_8]$.

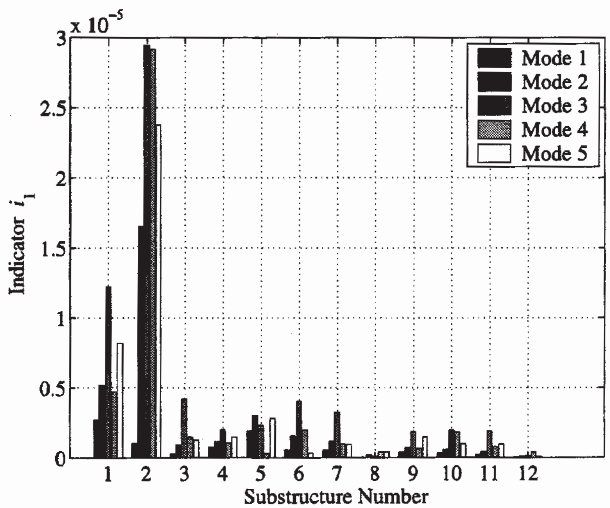

FIGURE 4 The indicator i_1 for Case 1. The indicator is calculated for the first five modes individually. $\tilde{\Phi}_0 = [\phi_8\phi_9]$.

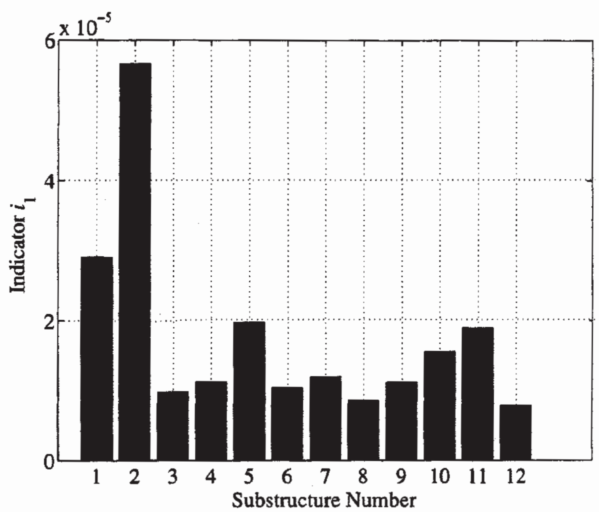

FIGURE 5 The indicator i_1 for Case 1. The indicator is calculated for the first five modes simultaneously. $\tilde{\Phi}_0 = [\phi_8\phi_9]$.

Figures 6 and 7 show the results for the i_2 indicator, where the measured global modes have been treated independently and in Figure 6 $\varphi_0 = \phi_8$, and in Figure 7 $\varphi_0 = \phi_9$. When mode 8 of the

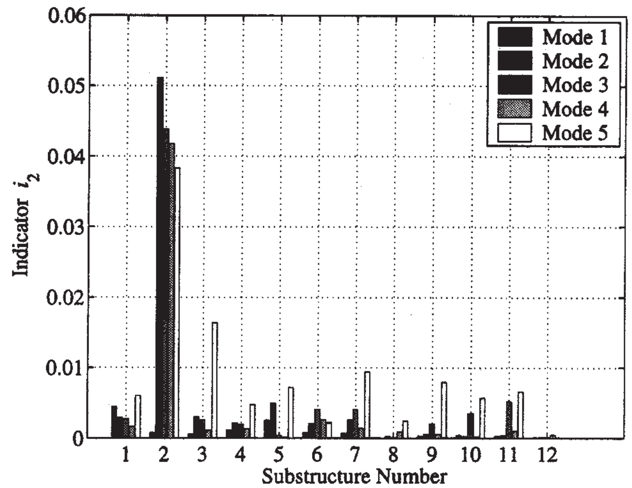

FIGURE 6 The indicator i_2 for Case 1. The indicator is calculated for the first five modes individually. $\tilde{\varphi}_0 = \phi_8$.

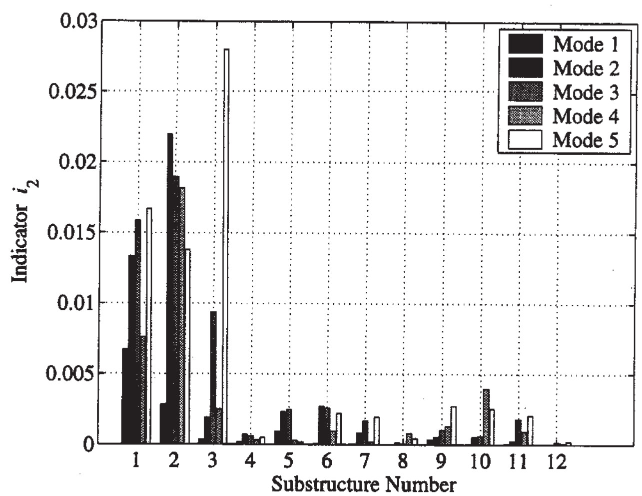

FIGURE 7 The indicator i_2 for Case 1. The indicator is calculated for the first five modes individually. $\tilde{\varphi}_0 = \phi_9$.

substructure is used the damage location is clearly identified, whereas the result is not so clear when mode 9 is used, although the general area of the damage has been located.

Case 2 Figures 8 to 10 show the results for Case 2, where substructures 2 and 6 have been damaged. Both the i_1 and i_2 indicators are able to locate the damage, although the choice of substructure modes used has been determined based on the experience of Case 1.

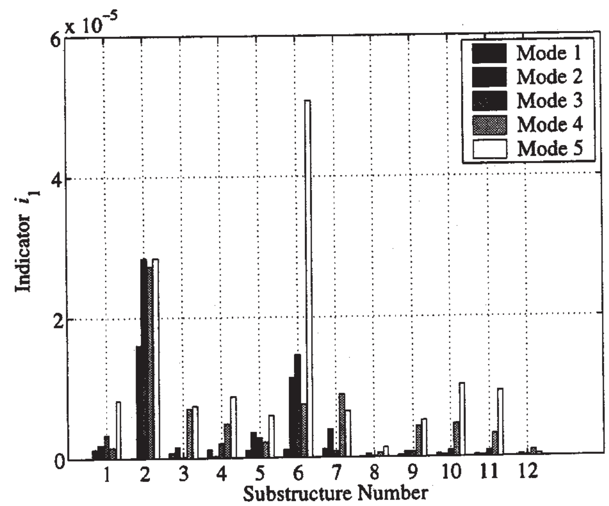

FIGURE 8 The indicator i_1 for Case 2. The indicator is calculated for the first five modes individually. $\tilde{\Phi}_0 = [\phi_8]$.

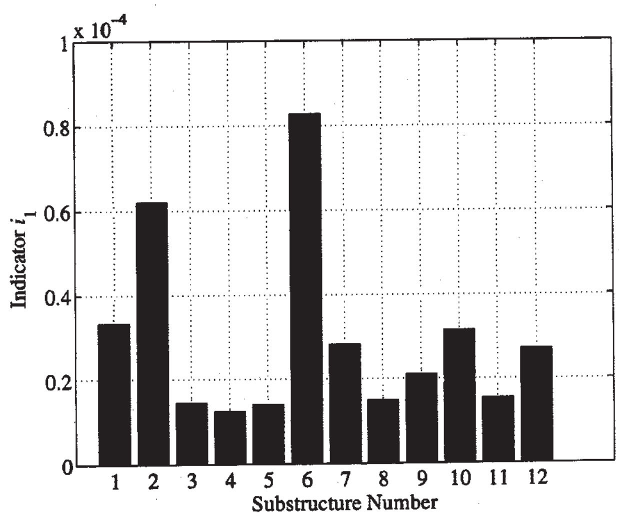

FIGURE 9 The indicator i_1 for Case 2. The indicator is calculated for the first five modes simultaneously. $\tilde{\Phi}_0 = [\phi_8\phi_9]$.

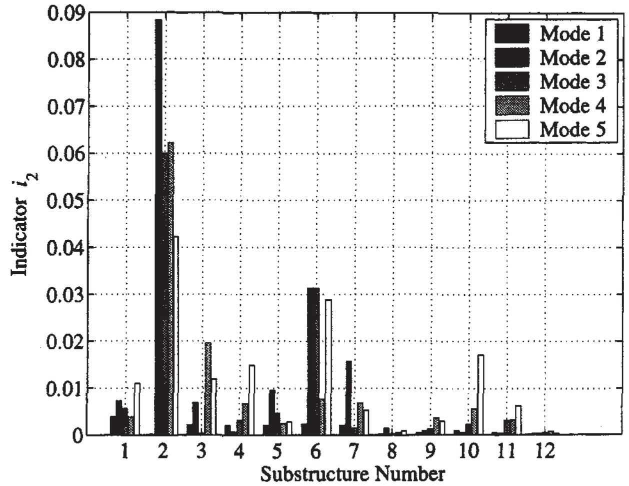

FIGURE 10 The indicator i_2 for Case 2. The indicator is calculated for the first five modes individually. $\bar{\varphi}_0 = \phi_8$.

4. CONCLUSIONS

Damage location procedures are proposed which use measured displacements from a structure and predictions from a mathematical model. Two damage location indicators have been derived based on the invariance of substructure mode-shapes to damage elsewhere, and the participation of higher substructure mode shapes in the measured displacements only when damage is present locally. It has been demonstrated that the procedures are capable of locating damage in a numerical example. However the success of the indicators depends to some extent on the choice of higher substructure modes used, and further work is continuing on how to determine the optimum set of modes for a given substructure.

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